The *DEMOGRAPHY-STATISTICS-INFORMATION TECHNOLOGY* Letter
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Demography 101: Population Growth Rates

From time to time people ask me "Demography 101" questions about basic concepts and methods. Growth rates exemplify a fundamental approach to explaining demographic phenomena and are an essential component of many demographic models. They have practical uses as well, such as interpolation and extrapolation of population estimates.

United States Population, 1790-2010

Figure 1 plots United States population from the first decennial census in 1790 to the most recent in 2010. It shows that population increased greatly, that the increase generally accelerated over time, and that the increase during the Great Depression decade of 1930–1940 was less than in surrounding decades.

The population was re-defined in the 1890 census to include Indians and Indian Territories and again in the 1960 census to include the newly created states of Alaska and Hawaii. The plot shows population under the old as well as the new

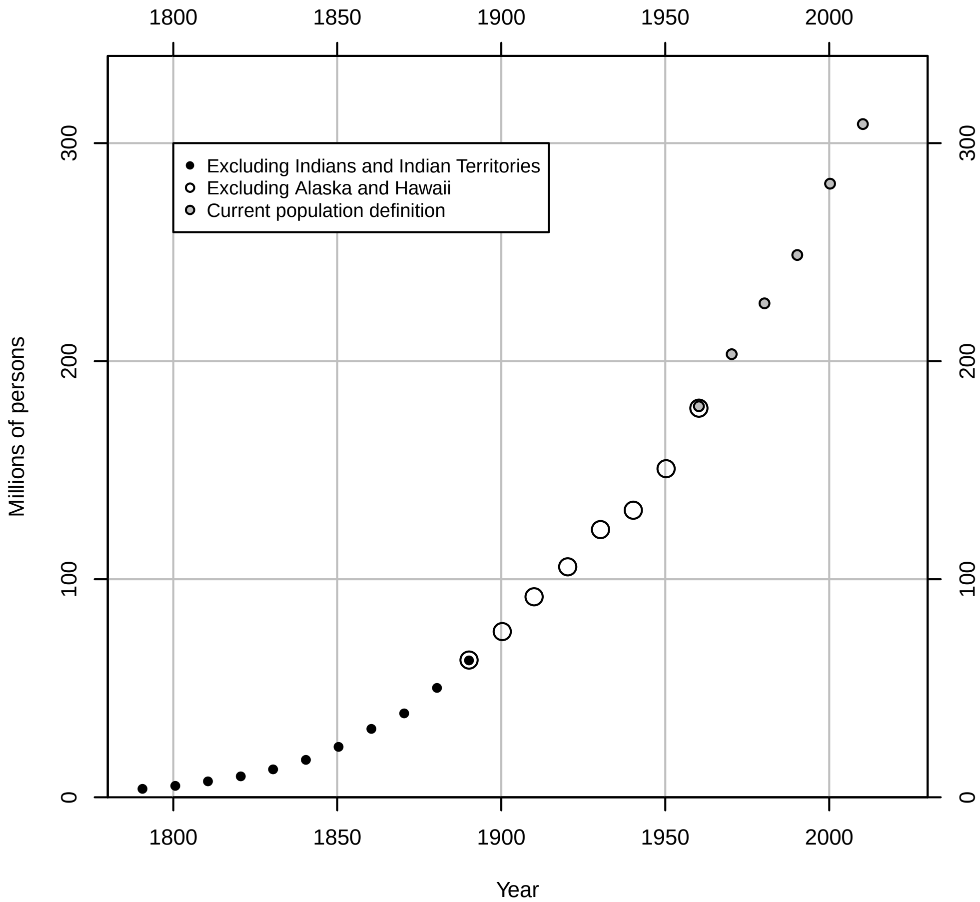

Figure 1. United States population, 1790-2010. Enumerated population at each decennial census. The population was re-defined twice, first prior to the 1890 census and again prior to the 1960 census. The plot shows population under the old as well as the new definition for these years. ***Source*** Table 1.

definition for these years.

Growth rates

Table 1 shows the population numbers plotted in Figure 1 together with the intercensal population growth and the intercensal growth rate. Growth rates are calculated as

$$\frac{ln(P_2/P_1)}{t}, \tag{1}$$

where P_1 is the time of the first census, P_2 the time of the second census, t the interval between the censuses, and ln natural logarithm. Given the 2000 and 2010 populations in Table 1, for example, $308,746/281,422 = 1.0971$, $ln(1.0971) = 0.0927$, and $0.0927/10 = 0.0927$, 10 years being the interval between censuses. The growth rate is 0.0927, but growth rates may be multiplied by 100 for presentation and referred

Table 1. United States population, population change, and growth rates, 1790-2010 (population numbers in thousands)

Year	Date	Current population definition	Excluding Alaska and Hawaii	Excluding Indians and Indian Territories	Decade	Inter-censal growth	Inter-censal growth rate (%)
2010	Apr 1	308,746	-	-	2000-10	27,324	0.93
2000	Apr 1	281,422	-	-	1990-00	32,712	1.24
1990	Apr 1	248,710	-	-	1980-90	22,164	0.93
1980	Apr 1	226,546	-	-	1970-80	23,334	1.09
1970	Apr 1	203,212	-	-	1960-70	23,889	1.25
1960	Apr 1	179,323	178,464	-	1950-60	28,626	1.74
1950	Apr 1	-	150,697	-	1940-50	19,028	1.35
1940	Apr 1	-	131,669	-	1930-40	8,894	0.70
1930	Apr 1	-	122,775	-	1920-30	17,064	1.50
1920	Jan 1	-	105,711	-	1910-20	13,739	1.39
1910	Apr 15	-	91,972	-	1900-10	15,977	1.91
1900	Jun 1	-	75,995	-	1890-00	13,047	1.88
1890	Mar 1	-	62,948	62,662	1880-90	12,792	2.27
1880	Jun 1	-	-	50,156	1870-80	11,598	2.63
1870	Jun 1	-	-	38,558	1860-70	7,115	2.04
1860	Jun 1	-	-	31,443	1850-60	8,251	3.04
1850	Jun 1	-	-	23,192	1840-50	6,129	3.07
1840	Jun 1	-	-	17,063	1830-40	4,202	2.83
1830	Jun 1	-	-	12,861	1820-30	3,223	2.88
1820	Aug 7	-	-	9,638	1810-20	2,398	2.86
1810	Aug 6	-	-	7,240	1800-10	1,932	3.10
1800	Aug 4	-	-	5,308	1790-00	1,379	3.01
1790	Aug 2	-	-	3,929	-	-	-

Sources For 1790-1990, Table 1, "United States - Race and Hispanic Origin: 1790 to 1990" in "Historical Census Statistics on Population Totals By Race, 1790 to 1990, and By Hispanic Origin, 1970 to 1990, For The United States, Regions, Divisions, and States", by Campbell Gibson and Kay Jung, U. S. Census Bureau Washington, DC 20233, September 2002. For 2000-2010, Table 1 of *Population Distribution and Change: 2000 to 2010*, March 2011, US Census Bureau. ***Notes*** Growth and growth rate for 1880-90 and 1950-60 calculated using current population definition. Growth rate is average annual growth rate. See text for explanation.

to as percentages. This explains the value 0.93 shown in upper right cell of Table 1.

Unfamiliarity with logarithms need not be an impediment to using formula (1). Calculations done using spreadsheet program will require entering a formula like `=LN(A1/A2)` in a cell, which will then show the the natural logarithm of the ratio of the numbers in cells `A1` and `A2`. Scientific calculators typically have a button that will produce natural logarithms.

When two censuses have the same reference date, the interval between them is the year of the second minus the year of the first. If the dates are different, the interval must be calculated as difference between the times of the two censuses. Dates may be converted to times by summing the day and the number of days in preceding months, dividing by 365 and, adding the result to the year. Leap years may be ignored. Expressing results to three digits after the decimal point makes it possible to recover the date from the time.

Comparing growth rates and population change

Figure 2 plots the intercensal growth rates. When plotting numbers that refer to a time period, rather than to a point in time, it is customary and appropriate to plot the number for a period at the midpoint of the period.

The growth rate plot shows fluctuations not apparent in Figure 1. Some of these may reflect inaccuracy in the counts, but others evidently reflect historical events—

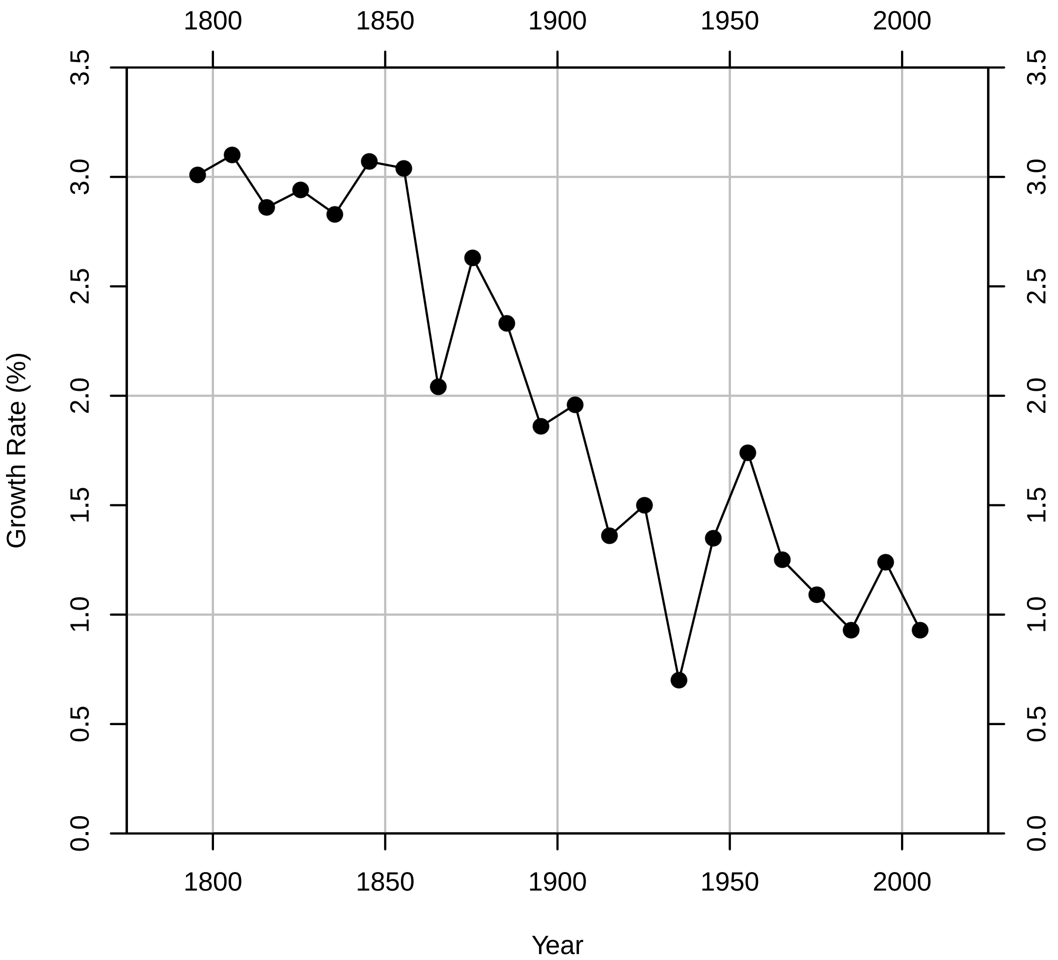

Figure 1. United States intercensal population growth rates, 1790-2010. ***Source*** Table 2.

most obviously the spikes down for the Civil War decade of 1860-70 and the Great Depression decade of 1930-40, the rapid rise to the Baby Boom decade peak for 1950-60, and the subsequence decline.

Figure 2 also gives a different impression of long term population trends. Figure 1 and Table 2 show that population increase is lowest during the early decades and highest during the final decades of the period.

Figure 2 shows the highest growth rates, about 3%, between 1790 and 1860, generally declining rates between 1860 and 1940, and the lowest rates between 1940 and 2010. Rates are more variable during the last period, but the typical level is about 1.1%.

Why population growth rates?

The comparison of Figures 1 and 2 suggests the value of looking at growth rates as well as population counts and population change, but it is useful to ask more generally why we calculate growth rates.

Table 1 shows that population increased by 1.38 million persons between 1790 and 1800 as compared with 27.3 million between 2000 and 2010. What explains this large increase? The answer is that growth increased because population increased. In 1790 there were 3.93 million persons. In 2000 there were 309 million persons. A larger population tends to more families, which tends to more children, which tends to more growth.

The word *tends* here is carefully chosen. There is no "law" at work. The tendency for larger populations to increase more is fundamental, but it is only a tendency—there are other influences at work. If one population is ten times the size of the other, the size difference will almost certainly dominate. If two populations, or the same population at different times, are similar in size, other influences may override the size difference.

In the familiar language of empirical social science, growth rates "control for" differences in population size. By doing so they improve our ability to discern the influence of other factors. That is why we see more history, more clearly, in Figure 2 than in Figure 1.

Mathematics of growth rates

Why the natural logarithms in Formula (1)? Consider percent increase of population over a time period,

$$100 \times \frac{P(t_2) - P(t_1)}{P(t_1)}, \tag{2}$$

where $P(t)$ denotes population size at time t and $t_2 > t_2$ are points in time.

Percent increase controls for the beginning population size, but not for length of the period. Nor does it control for the influence on growth of persons added to the population during the period. This is the "compounding" effect familiar from interest rate calculations and from some discussions of population growth.

Dividing percent increase by the length of the time period will evidently provide some control for length of the time period. Moreover. since shorter periods mean less compounding, control should improve for shorter time periods.

Dividing (2) by $P(t_1)$ and putting $t_1 = t$ and $t_2 = t + \Delta$ gives

$$\frac{P(t+\Delta) - P(t)}{P(t)\Delta}, \tag{3}$$

which students of elementary calculus will recognize as an expression whose limit as Δ approaches zero is the slope of the tangent line to $P(t)$ at time T divided by $P(t)$, i.e., the derivative $P'(t)$ of $P(t)$ at t divided by $P(t)$.

This brings us to a formula for the growth rate at exact time t, sometimes called an "instantaneous" growth rate.

$$r(t) = P'(t)/P(t) \tag{4}$$

Solving this differential equation gives

$$P(t+n) = P(t)exp\Big\{\int_t^{t+n} r(t)dt\Big\}, \tag{5}$$

where the "boundary condition" $P(t)$ is given. The method of solution may be found in any introductory calculus text.

If $r(t)$ is constant with value r between times t and $t+n$, (5) becomes

$$P(t+n) = P(t)e^{rn}. \tag{6}$$

Formula (1) results from dividing both sides of this equation by $P(t)$ and taking natural logarithms. The growth rate calculated from Formula (1) may accordingly be referred to as the "average annual instantaneous" growth rate for the intercensal period.

Continuous formalism in demographic analysis

The appearance of the differential and integral calculus in this discussion of population growth rates should surprise us. The calculus was invented to solve problems in physics involving phenomena that were self-evidently continuous. Demographic phenomena are self-evidently discrete. Why the continuous mathematics?

The instantaneous growth rate formula (4) makes sense only on the assumption that $P(t)$ is a smooth function. The true population size function is necessarily discontinuous, jumping up or down when persons enter or leave the population. Certainly we may imagine constructing a smooth function that approximates the true function, and the approximation can be more than adequate unless the population is very small. But again, why would we do this?

The answer turns out to be that this continuous formalism is convenient for developing and expressing population models. The models are of immense practical utility, and the formalism has been used for over a century by essentially everyone who has written on the subject.

Conclusion

Demographic phenomena reflect three kinds of influences, biological, behavioural, and formal demographic. Age patterns fertility and mortality reflect human biology and human behaviour. The influence of population size on population growth is a simple example of a formal demographic influence.

Formal demographic influences are a large part of what distinguishes demography from biology and sociology (and economics, geography, anthropology, statistics, ...). Population growth rates bring us a bit closer to the human behaviour and biological influences on population growth. That is why they are important. But they are only the first step of a longer journey.

Resources

The sources for Table 1 are available at census.gov (they may be found most simply by searching all or part of the titles). ***Downloads*** us-population-growth-1790-2000.xls is a spreadsheet containing population numbers, calculations and plots for Table 1 and Figures 1 and 2). table1.R contains R code for reading the spreadsheet data into R using XLConnect, produced by MiraiSolutions. figure1.R and figure2.R contain R code for creating Figures 1 and 2. They illustrate techniques for fine control over plots and for producing high quality output for publication.

Publication note

Letters are scheduled for publication at two month intervals from mid-January through mid-November, but I fell behind schedule this year under pressure of other work.

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