ESTIMATING INFANT MORTALITY RATES FROM CHILD SURVIVORSHIP DATA BY AGE OF MOTHER

by Griffith Feeney

Editor's Note: Griffith Feeney, Research Associate and Assistant Director for Graduate Study at the East-West Population Institute, is the author of several previous technical notes in this newsletter. At the fourth Population Census Workshop-Conference in Honolulu last May, Feeney gave a paper on the relationship of census measurement of nuptiality to child mortality estimation. Participants at the conference suggested that the technique be presented in a form that would facilitate practical application. Subsequent research by Feeney has uncovered a simple short-cut method for deriving infant mortality estimates, and he describes the method below.

Several methods are currently used to estimate mortality from child survivorship data. Most of these methods assume that mortality has been constant prior to the census, however, and those few methods that allow for changing mortality require that the rate of mortality change be known. Recent research at the East-West Population Institute has resulted in the development of a new method that allows for changing mortality and does not require knowledge of the rate of change. This new method uses the same data as existing methods, is simple to apply, and provides estimates of the infant mortality rate for approximately 15 years prior to the census or survey in which the data were collected. This article describes the new method and gives several examples of its application.[1]

Child survivorship data by age of mother

The basic child survivorship information provided by a census or survey is exemplified by Table 1, which gives data from the 1973 Census of the Gilbert and Ellice Islands Colony. The first step in analyzing these data is to calculate the proportions of deceased children for women in each age group. The number of deceased children is obtained as the difference between children born and children surviving; division by the number of children born gives the proportion deceased. This leads to the following figures.

15–19	0.114
20–24	0.107
25–29	0.122
30–34	0.155
35–39	0.190
40–44	0.226
45–49	0.251

These proportions of deceased children are not in themselves useful indices of mortality, however. The proportion deceased for women aged 45–49 is more than twice the proportion for women aged 15–19, for example, but it does not follow that the children of the older women experienced higher mortality. Since the children of the older women were born longer ago, they have been exposed to the risk of mortality over a longer period and it is to be expected that, if the two groups experienced the same mortality risks, the proportion of children deceased will be greater for these children than for the children of the younger women.

It may be that the children of the older women did experience higher mortality. Indeed, if mortality has been declining, we would expect this to be the case, for almost all the children of the younger women were born during the five years immediately prior to the census and are therefore exposed only to the mortality risks of the recent past, whereas most of the children of the older women were born longer ago and therefore experienced the mortality risks of earlier periods. But it is impossible to draw a conclusion one way or another on the basis of the proportions deceased alone. It is necessary first to consider the exposure to mortality of each group of children.

Table 1 Child survivorship data by age of mother: Gilbert and Ellice Islands Colony, Census of 8 December 1973

Age group	Number of women	Children born	Children surviving
15–19	2,980	464	411
20–24	2,643	2,819	2,516
25–29	1,871	4,575	4,016
30–34	1,739	6,935	5,862
35–39	1,484	7,645	6.191
40–44	1,296	7,657	5,924
45–49	1,230	7,592	5,685

SOURCE: *Gilbert and Ellice Islands Colony. Report on the 1973 Census of Population.* Volume 1. *Basic information.* Total women and children born from Table 32, page 178; children surviving from Table 33, page 180. Number of women excludes women not reporting number of children born.

To calculate the period of exposure we must consider, for the group of all children born to women in a given age group, how many of these children were born during the first year prior to the census, how many were born during the second year prior to the census, and so forth. This distribution of children born by year of birth depends on the pattern of fertility in the population as well as on the age group of their mother. In a population in which childbearing begins very late, for example, most children to women aged 15–19 will have been born not more than one or two years before the census, whereas, in a population in which childbearing begins at a very early age, a much smaller proportion will have been born so recently.

The new method takes these considerations into account by using an estimate of the mean age at childbearing for the population. This mean age may be estimated as follows. We first calculate the figures for mean number of children born from the data in Table 1.

15–19	0.156
20–24	1.067
25–29	2.445
30–34	3.988

The ratio of the first two parties is 0.156 ÷ 1.067 = 0.146. Locating this value in the left column of Display 1, we see that the displacement of the mean age at childbearing from the age which separates the two five-year groups—age 20 in this example—is +9. Hence 9 is added to 20 to give an estimated mean age at childbearing of 29 years. The same procedure may be repeated with the mean parities for the age groups 20–24 and 25–29; the mean parity ratio is 1.067 ÷ 2.445 = 0.436, and this corresponds to a displacement of +4 from age 25; hence the mean age at childbearing is estimated to be 25 + 4 = 29 years, the same value as before. Re-

1 Brass, et al. (1968:104–120), Sullivan (1972), and Trussell (1975) give methods assuming constant mortality. Brass (1975), Sullivan (1974), and Kraly and Norris (1976) consider changing mortality. The new method presented here follows closely the original approach developed by Brass (1953, 1961, 1968) and draws also on the work of Macura (1972).

Display 1 Estimation of the mean age at childbearing from mean parity ratios for successive five-year age groups

1000 X Mean parity for women aged $x - 5$ to x / Mean parity for women aged x to $x + 5$	Displacement of mean age at childbearing from x
063–110	+10
111–167	+ 9
168–230	+ 8
231–293	+ 7
294–353	+ 6
354–409	+ 5
410–461	+ 4
462–508	+ 3
509–552	+ 2
553–593	+ 1
594–630	0
631–665	− 1
666–697	− 2
698–728	− 3

peating the procedure once again for the age groups 25–29 and 30–34 yields a mean parity ratio of 0.613 and a mean age of childbearing of 30 years. The final estimate of the mean age at childbearing is taken to be the mean of these three results, (29 + 29 + 30) ÷ 3 = 29.3 years.

Estimating infant mortality rates

The new method provides an estimated infant mortality rate and a number of "years-prior-to-census" for each age group. The infant mortality rate is expressed as infant deaths per thousand live births, and the years-prior-to-census figure indicates the point in time to which this estimated rate applies.[2] The years-prior-to-census figure dates this estimate, indicating that it applies as of the given number of years prior to the census date. Both figures are obtained by performing the calculations indicated in Display 2. The symbols M and Q represent, respectively, the mean age at childbearing and the proportions of deceased children. Using data from the 20–24 age group of the Gilbert and Ellice Islands Colony, for example, one enters M = 29.3 and Q = 0.107 in the first line of Display 2 and obtains an estimated infant mortality rate of $[-44.7 + (30.5 \times 29.3)] \times 0.107 - 2.6 = 88$ infant deaths per thousand births, and a number of years prior to the census of $11.8 - (0.325 \times 29.3) - (0.17 \times 0.107) = 2.3$ years. Similar calculations for successive age groups give the following infant mortality rates and years-prior-to-census figures.

Age group	IMR	YPC
20–24	88	2.3
25–29	86	4.1
30–34	100	6.2
35–39	115	8.5
40–44	128	11.7
45–49	128	15.0

These estimates are most readily appraised by plotting them (see Figure 1). The plotted points in this case suggest an approximately linear decline in the infant mortality rate, with deviations from linearity for the estimates based on the oldest and youngest age groups. The apparent increase in

2 The estimated rate refers to an exact point in time, not to a time period. If the force of mortality for time t at exact age a is denoted $\mu(a,t)$, the quantity estimated is $1 - \exp[-\int_0^1 \mu(a,t)da]$, the infant mortality rate at time t exactly.

Display 2 Estimation of infant mortality rates from proportions of deceased children among children born to women in five-year age groups, given the mean age at childbearing

Age group	Infant mortality rate	Years prior to census
20–24	$(-44.7 + 30.5M)Q - 2.6$	$11.8 - 0.325M - 0.17Q$
25–29	$(294 + 14.9M)Q - 2.9$	$16.5 - 0.424M + 0.16Q$
30–34	$(357 + 10.4M)Q - 2.8$	$20.6 - 0.494M + 0.77Q$
35–39	$(362 + 9.77M)Q - 7.8$	$24.9 - 0.556M + 0.80Q$
40–44	$(282 + 11.0M)Q - 8.5$	$30.1 - 0.633M + 0.87Q$
45–49	$(216 + 11.1M)Q - 7.5$	$33.4 - 0.641M + 1.58Q$

NOTE: The proportion of deceased children is represented by Q, the mean age at childbearing by M.

mortality during the five years preceding the census is probably spurious. The relatively high estimate for children born to women aged 20–24 might be due to errors in the census data or to differentially high mortality for children of young mothers. The graph in Figure 1 also suggests that the decline in mortality began about twelve years prior to the census. This may be the case, but the relatively low mortality rate for the children born to women aged 45–49 might also reflect a failure of these women to report some deceased children in response to the question on children ever born. *In view of these considerations, the estimates derived from the 20–24 and the 45–49 age groups should perhaps be accorded less weight than the estimates derived from the intermediate age groups.*

Practical considerations

The various calculations involved in this method of estimation are most conveniently executed on the work form shown in Table 2, which contains entries for the Gilbert and Ellice Islands Colony example just discussed. The figures for

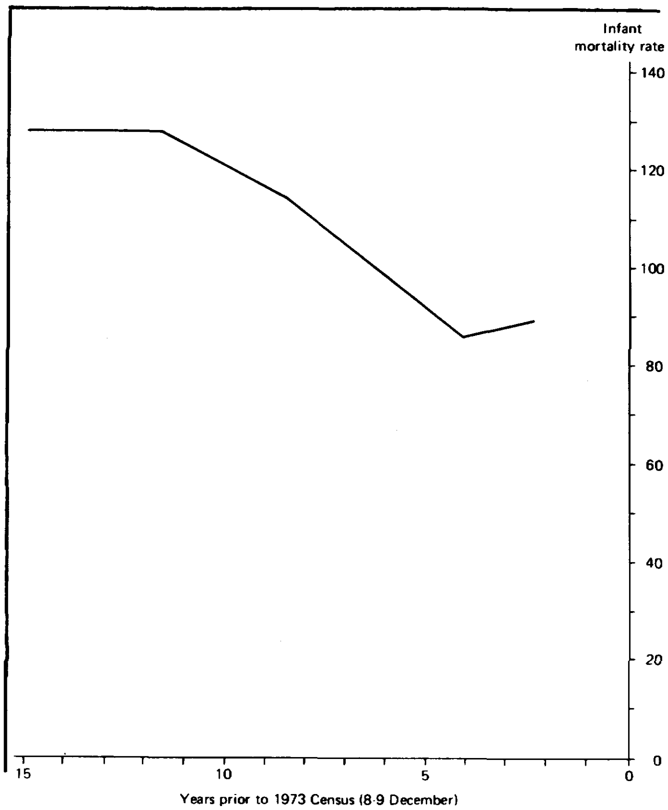

Figure 1 Estimated infant mortality rates: Gilbert and Ellice Islands Colony, 1968 Census

mean number of children born and proportion of deceased children are calculated first, and, if a calculator with one memory location is used, the number of children born may be entered once and stored for use in both calculations. Although figures for the mean number of children born for the age groups beyond 30–34 are not usually required, they provide useful background information and may as well be calculated and entered on the form.

The estimated mean ages at childbearing are entered next. There will usually be three estimates. The first is derived from the ratio of the mean number of children born to women aged 15–19 to the mean number of children born to women aged 20–24. This ratio is entered in the row identified with the 15–19 age group. The second estimate is derived from the corresponding ratio for the 20–24 and 25–29 age groups and entered in the 20–24 row. The third is derived from the ratio for the 25–29 and 30–34 age groups and entered in the 25–29 row. In some cases, however, one of these mean parity ratios may fall outside the range of the lefthand column of Display 1, in which case no estimate is obtained from that mean parity ratio. One may obtain only two estimates, or one may obtain three estimates based on the age groups in the range 20–39 instead of the usual 15–34 range. In any case, the procedure for obtaining the final estimate of *M* for use in the calculation

Table 2 Work form for estimating child mortality rates from child survivorship data by age of mother: Data for Gilbert and Ellice Islands Colony, Census of 8 December 1973

Age group	Mean number of children born (MCB)	Mean age at childbearing (MAC)	Proportion of deceased children (Q)	Infant mortality rate (IMR)	Years prior to census (YPC)
15–19	0.156	29	0.114	–	–
20–24	1.067	29	0.107	88	2.3
25–29	2.445	30	0.122	86	4.1
30–34	3.988	–	0.155	100	6.2
35–39	5.152	–	0.190	115	8.8
40–44	5.908	–	0.226	128	11.7
45–49	6.172	–	0.251	128	15.0
		(*M* = 29.3)			

SOURCE: Table 1.

of the infant mortality rate and years-prior-to-census values is always the same. One determines estimates of the mean age at childbearing for as many mean parity ratios as lie within the range of the lefthand column of Display 1 and calculates the average of these values.

The infant mortality rates and the years-prior-to-census figures are entered last using the formulas specified in Display 2. The value of *M* is the same for all age groups. Note that no infant mortality estimate is obtained from the data for the 15–19 age group. This last step is the most tedious computationally, and care must be taken to observe the signs of the various terms in Display 2. Once the procedure is familiar it takes about 15 minutes to work through the entire worksheet using an automatic calculator.

Estimates derived from successive censuses

In the case of the Gilbert and Ellice Islands Colony, child survivorship data are available for the 1968 Census as well as for the 1973 Census, and since the time periods represented by the estimates derived from these two censuses overlap, the estimates may be checked for consistency. Table 3 shows the basic data for 1968, Table 4 the derived

Table 3 Child survivorship data by age of mother: Gilbert and Ellice Islands Colony, Census of 5 December 1968

Age group	Number of women	Children born	Children surviving
15–19	2,778	553	487
20–24	1,938	2,659	2,305
25–29	1,747	5,433	4,593
30–34	1,470	6,639	5,339
35–39	1,468	8,300	6,472
40–44	1,061	6,498	4,924
45–49	1,014	6,000	4,238

SOURCE: *A Report on the Results of the Census of Population: 1968* (Gilbert and Ellice Islands Colony). Number of women from Table 6, page 184. Excludes women not reporting number of children born. Children born calculated from Table 6, page 184, assuming 15 children per woman for 15+ parity women and similarly for number of surviving children (Table 7, page 188). It is unnecessary to make any adjustment to this figure in view of the small number of women in the open-ended groups. See Feeney (1976) for a general discussion.

infant mortality estimates. To compare the two sets of estimates it is necessary to transform the years-prior-to-census values into calendar time. This is most readily accomplished by converting conventional dates from month, day, and year to decimal expressions. The general procedure consists of adding the total number of days in the months preceding the given month to the given day less one half, dividing this result by 365, and adding the resulting decimal fraction to the given year. Display 3 gives dates corresponding to each tenth of a year. The 1968 Census, for example, was taken

Display 3 Conversion of calendar dates into tenths of a 365-day year

Calendar dates	Tenths of year
1 January–18 January	0.0
19 January–24 February	0.1
25 February–1 April	0.2
2 April–8 May	0.3
9 May–13 June	0.4
14 June–20 July	0.5
21 July–25 August	0.6
26 August–1 October	0.7
2 October–6 November	0.8
7 November–13 December	0.9
14 December–31 December	1.0

as of 5 December. Locating this date in the left-hand column of Display 3 gives a calendar time of 1968.9 years. The estimate of 108 per thousand 2.4 years prior to the census (Table 4) thus corresponds to the calendar time 1968.9 - 2.4 = 1966.5 years.

Following this procedure for the estimates of both censuses (Tables 2 and 4) yields the following results:

Census of 1968		Census of 1973	
Calendar time	Rate	Calendar time	Rate
1966.5	108	1971.6	88
1964.5	109	1969.8	86
1962.3	126	1967.7	100
1959.8	134	1965.1	115
1956.8	136	1962.2	128
1953.4	150	1958.9	128

Table 4 Infant mortality rates estimated from child survivorship data by age of mother: Gilbert and Ellice Islands Colony, Census of 5 December 1968

Age group	Mean number of children born (MCB)	Mean age at childbearing (MAC)	Proportion of deceased children (Q)	Infant mortality rate (IMR)	Years prior to census (YPC)
15–19	0.199	29	0.119	–	–
20–24	1.372	29	0.133	108	2.4
25–29	3.110	28	0.155	109	4.4
30–34	4.516	–	0.196	126	6.6
35–39	5.654	–	0.220	134	9.1
40–44	6.124	–	0.242	136	12.1
45–49	5.917	–	0.294	150	15.5
		(M = 28.7)			

SOURCE: Table 3.

The comparison of these two series of estimates is facilitated by plotting them, as shown in Figure 2. Inspection of Figure 2 shows that the two series overlap during the period 1960–1966, and although certainly not coincident during this period, the two series are generally consistent. Both series include estimates for approximately 1960, 1962.5, and 1965, and averaging the 1968 rates for 1959.8, 1962.3, and 1964.5 gives 123 per thousand, as compared with 124 per thousand, the average of the 1973 rates for 1958.9, 1962.2, and 1965.1. This comparison suggests that the overall levels indicated by the series are reasonably accurate. This overall level may be summarized by averaging both the infant mortality rates and the years-prior-to-census values for the four intermediate estimates from both censuses. This gives an infant mortality rate of 117 infant deaths per thousand births for 1963.5.

The rate of mortality decline indicated by the estimates from a single census may reflect such spurious elements as relatively high mortality for the children of young mothers or underreporting of deceased children. In the extreme case, this could render the estimates useless for determining either the level or the trend of mortality. If data are available from two successive censuses, however, rates of mortality decline may be calculated from the estimates for corresponding age groups, and, so long as errors and biases are the same for both censuses, the rates of decline will be accurate. Thus for example the 1968 Census data for age 20–24 give an estimate of 108 per thousand in 1966.5 and the 1973 Census data for this same age group give an estimate of 88 per thousand in 1971.6. Combining these estimates yields an estimated rate of decline of (108 – 88) ÷ (71.6 – 66.5), or 3.9 per thousand per year. The estimates based on successive age groups are

Age group	Rate of decline
20–24	3.9
25–29	4.3
30–34	4.8
35–39	3.6
40–44	1.5
45–49	4.0

The irregularity of these estimates suggests "noise" in the data which may be attenuated by aggregation over several age groups. We simply average the estimated infant mortality rates and years-prior-to-census figures for two or more groups. By combining the data for the 20–24 and 25–29 groups, for example, we obtain a level of 108.5 per thousand for 1965.5 from the 1968 Census, a level of 87.0 per thousand for 1970.7 from the 1973 Census, and hence a rate of decline of 4.1 per thousand per year. The estimates of the rate of decline of the infant mortality rate obtained in this way may be examined for patterns in many ways. In general, however, the best overall estimate of the rate of mortality decline is probably obtained by averaging all the estimates obtained from each census and calculating the rate of decline between these two figures. This yields an infant mortality rate of 127.2 for 1960.6 from the 1968 Census, an infant mortality rate of 107.5 for 1965.9 from the 1973 Census, and a rate of decline of 3.7 infant deaths per thousand births per year.

The estimated level of infant mortality, 117 per thousand at 1963.6, and the estimated rate of decline, 3.7 per thousand per year, together define the level of mortality at any specified time within the period spanned by the estimates. To find the level at the time of the 1968 Census, for example, 1968.9, we simply calculate 117 – 3.7 × (68.9 – 63.6) = 97.4 infant deaths per thousand births. These two estimates also define the average level of infant mortality over any time period, which is simply the average of the levels at the beginning and end of the period.[3] Suppose for example that we wanted to compare the estimates with registration data for the years 1965–69. We would calculate a rate of 117 – 3.7 × (65.0 – 63.6) = 111.8 per thousand for 1965.0, a rate of 117 – 3.7 × (70.0 – 63.6) = 93.3 per thousand for 1970.0, and hence a rate of 102.6 per thousand for the period 1965–69.

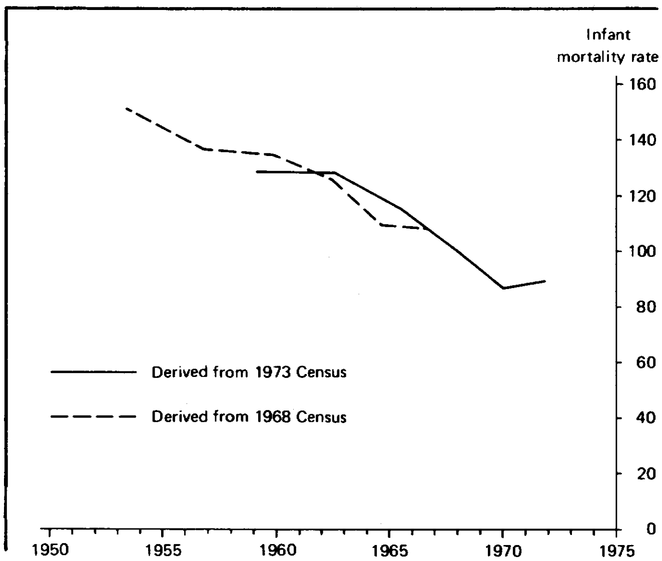

Figure 2 Estimated infant mortality rates: Gilbert and Ellice Islands Colony, 1968 and 1973 Censuses

Theory behind the new method

It is my no means obvious why the formulas in Display 2 should provide estimates of the infant mortality rate. The

3 The assumption is made that births are uniformly distributed during the period in question. Departures from this assumption will in most cases have a negligible effect on the result. Where a substantial effect is suspected, an effort should be made to estimate the actual time distribution of births and apply the linear trend of infant mortality rates to this distribution to obtain the average level for the period.

method is based on the equation $Q = 1 - \Sigma_j c_j p_j(\omega,r)$ where Q denotes the proportion of deceased children born to women in a particular age group, c_j denotes the proportion of this group of children who were born in the j-th year prior to the census, and $p_j(\omega,r)$ denotes the proportion of children born during the j-th year prior to the census who would survive to the time of the census under the following suppositions: (1) the infant mortality rate was ω at the time of the census and had been declining at a constant rate r in the years prior to the census; (2) there was no differential mortality by age of mother; and (3) the life table representing the mortality experience each year prior to the census is included in a known, one-parameter model life table family. The values of c_j may be estimated from the mean age at childbearing; hence this equation may be rewritten as $Q = 1 - \Sigma_j c_j(M) p_j(\omega,r)$.

For any specified values of Q and M, one may determine combinations of values of ω and r that satisfy this equation, and each such pair of values determines a certain linear trend of mortality consistent with the given proportion Q of deceased children. It turns out empirically that these consistent linear mortality trends intersect, to a very close approximation, at a single point. The coordinates of this point give the estimated infant mortality rate and years-prior-to-census values. The formulas in Display 2 were obtained by first tabulating the coordinates corresponding to a range of values of Q and M and then fitting the tabular values by a simple, mathematical formula.

Application of the new method

Estimates obtained by the application of the new method must be interpreted with due regard for errors in the data, theoretical assumptions of the method, and the approximate character of the formulas in Display 2. Experience with child survivorship data from many different censuses shows that no single pattern or level of error can be expected in all circumstances. In some cases the quality of response to the census questions is very good; in others it is so poor that the data may be useless for mortality estimation. In some cases biases in the proportions of deceased children appear to increase sharply with age, whereas in other cases the evidence suggests a bias independent of age. The primary device for detecting errors in the data is the comparison of independent sources of information, including the comparison of estimates from successive censuses. If underreporting of deceased children increases sharply with the age of the woman, for example, the rates estimated from a later census will be substantially below those from an earlier census.

Even if the data were perfect, errors may be introduced into the estimates by the assumptions entailed by the method. Principal among these are the linear trend of infant mortality rates, the conformity of the population mortality schedules to a model life table family, and the absence of differential mortality by age of mother. In practice, of course, none of these assumptions will ever be precisely correct. The question is not whether they are valid but the extent to which they are valid and the effects of deviations from the assumptions on the final estimates. The estimates themselves indicate the extent to which the linearity assumption is valid by the extent to which the estimated infant mortality rates fall along a straight line. The estimates plotted in Figures 1 and 2, for example, suggest no substantial deviation from the linearity assumption. Differential mortality by age of mother may be evaluated to some extent by examining the pattern of the estimated infant mortality rates according to the age group from which they are derived. Thus for example the relatively high rate derived from the 20–24 age group in the 1973 Census (Figure 1) suggests a rise in infant mortality prior to the census. If circumstances in the population give no reason to expect such a rise, however, it might be attributed to relatively high infant mortality of children of very young mothers. If possible, such a conclusion should be reinforced by supplementary data.

Errors due to use of the approximate formulas in Display 2 may of course be eliminated by using the direct calculation procedure described briefly in the preceding section. Direct calculation may require as many as half a million basic arithmetic operations for a single age group, however; hence electronic computation is an absolute necessity. A Fortran program for this purpose is being developed at the Population Institute and should be ready for general distribution within a year. Research is also proceeding on more general and precise approximate methods for hand calculation as well as on systematic application of the procedure to available data on child survivorship. □

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