INDIRECT ESTIMATION OF PERIOD PARITY PROGRESSION RATIOS FROM TIME SERIES OF BIRTHS DISTRIBUTED BY ORDER

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The indirect estimation procedure detailed here was first described in Feeney (1985). It was subsequently applied to Japan in Feeney (1986), to the United States in Feeney (1988), to Taiwan in Feeney (1991), and to East Germany in Feeney and Lutz (1991:177-181). This note gives a more detailed exposition of the procedure than has appeared to date, including a detailed numerical illustration for the Taiwan data analyzed in Feeney (1991). Familiarity with the general ideas of parity progression measures is assumed. See Feeney and Wang (1991) for a detailed exposition.

The data required consists of a time series of annual numbers of births distributed by birth order. This series should be at least ten years long, for it will usually be necessary to discard the first five or so years of estimates. We also need (i) a standard schedule of progression rates for progression for each progression, first to second birth, second to third birth, and so on, and (ii) an open birth interval distribution for women of each parity at the beginning of the first year in the birth order data series.

Determining the Standard Schedules. While the choice of the standard schedule of progression rates is not very critical, the general shape should conform to the shape of the schedules experienced by the population. Information on these schedules will often be available from birth history fertility surveys. Because the estimation procedure is quite robust against variations in the shape of the schedules, however, an approach based on models will give results very nearly as good with substantially less effort.

When levels of progression to next birth are high (close to one), the parity progression rates increase with increasing during to a level of about one half and remain more or less at this level through high durations. This pattern reflects a situation in which all women are exposed to the risk of progressing to a next birth, and in which essentially all do so within about ten years of having their previous birth.

When levels of progression to next birth are low (one half or lower), parity progression rates increase with duration for the first several years, reaching a high of less than one half, and then decrease as we move to higher durations. The lower the level of progression to next birth, the lower the peak, and the more rapidly rates decline from this peak.

These patterns are consistent with a model according to which the birth interval distribution for women who progress to a next birth is identical no matter what the level of progression to next birth. It is heuristically useful, though not necessarily correct, to think of women having an i-th birth during any period as falling into two groups, one group that will progress to next birth according to a given model birth interval distribution, and another group that will never progress to a next birth. The latter women remain in the denominators of the parity progression rates through all durations in parity and so pull rates of progression down at higher durations.

To derive a standard schedule corresponding to all women progressing to next birth, we observe a series of empirical schedules of parity progression rates, compute the corresponding normalized birth interval distributions, and then calculate the schedules of parity progression rates corresponding to these normalized distributions. These transformed schedules should rise to a level of about one half after three or four years duration in parity and then remain approximately level thereafter.

These calculations have been done for data for several countries, and the resulting transformed schedules averaged to give a model schedule corresponding to (nearly) universal progression to next birth. The values of this standard schedule are shown in the 'Model Rates' column of Table 1.

Progression from first to second birth is generally fairly high, but still substantially less than one. In the case of Taiwan, a simple overall estimate of progression from first to second birth may be obtained as the total number of second births during 1975-1988 divided by the total number of first births during the same period. This gives a parity progression ratio of 0.837. Table 1 shows how to compute a schedule of parity progression rates corresponding to this parity progression ratios from the model schedules just described. We use the resulting schedule as the standard schedule for progression from first to second birth.

Progression from second to third birth will generally be lower, whence a standard schedule with a different shape is in order. Again we estimate an overall parity progression ratio by dividing total third births during 1975-1988 by total second births during the same period and use this value to compute a standard schedule for progression from second to third birth. The same procedure is used for all higher order progressions.

<u>Determining the Initial Open Birth Interval Distributions</u>. A characteristic of the estimation procedure is that the impact of the initial open birth interval distribution is very strong for the

first few years but declines rapidly, with little effect beyond five years and essentially none after ten years (Feeney 1986:1924). If we have a reasonably long birth order series, then, and are willing to loose the first five to ten years of estimates, a rough estimate of the initial open birth interval distribution will suffice.

The simplest way to estimate an initial open birth interval distribution is to assume a stationary population with numbers of first (or higher order) births equal to the observed number in the first year of the series and survivorship, i.e., retention in parity, defined by the standard schedule of parity progression rates for the given progression. Table 2 shows this calculation for parity one women at the beginning of 1975 in Taiwan.

The Estimation Procedure. The several inputs to the indirect estimation of period parity progression ratios for progression from first to second birth have now been assembled. The three elements required are (i) the time series of annual numbers of first and second births, for 1975-1988 for Taiwan in the case of Feeney (1991); (ii) the standard schedule of parity progression ratios for progression from first to second birth; and (iii) the initial open birth interval distributions.

The estimation procedure for higher order progressions is the same, so we may illustrate using progression from first to second birth only.

Table 3 shows two cycles of the estimation procedure. The standard rates are shown for reference in the column at left. These are applied to the observed number of 1975 first births and the initial open birth interval distribution (OBID) of parity one women at the beginning of 1975 to obtain a projected number of second births for 1975.

This projected number of second births is higher than the observed number of second births, indicating that the standard rates are too high for this year. We therefore multiply them by the ratio of observed to projected second births to obtain estimated parity progression ratios for the year, shown in the last column under the '1975' heading. The estimated period parity progression ratios for 1975 is then calculated from these rates.

Finally, we use the estimated rates to project an open birth interval distribution for the beginning of 1976, which will be used in the next cycle of the estimation procedure. The projection involves subtracting out women who have a second birth and incrementing duration in parity by one. Note that mortality is ignored in this calculation. While mortality could be incorporated, it would be a nuisance to do so, and there would be essentially no effect on the results.

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Table 1. Illustration of a Model for Birth Probabilities Specific for Parity and Duration in Parity

DIP	Model Rates	ACUMl	BIDl	BID2	DCUM2	Output Rates
E	.005	.0050	.0050	.0042	1.0000	.0042
0	.170	.1742	.1692	.1425	.9958	.1431
1	.335	.4508	.2766	.2330	.8533	.2731
2	.500	.7254	.2746	.2313	.6203	.3729
3	.500	.8627	.1373	.1157	.3890	.2974
4	.500	.9314	.0687	.0579	.2733	.2119
5	.500	.9657	.0343	.0289	.2154	.1342
6	.500	.9828	.0171	.0144	.1865	.0772
7	.500	.9914	.0086	.0072	.1721	.0418
8	.250	.9936	.0022	.0019	.1649	.0115
9	0	.9936	0	0	.1630	0

Notes

The inputs to the calculation are the model rates shown at far left and a parity progression ratio, taken as 0.837 in this example. The output of the calculation is a set of rates yielding the given parity progression ratio, calculated from the model rates on the assumption that the birth interval distributions corresponding to the two schedules are identical.

DIP: Duration in parity. "E" denotes women having first birth during given year, "O" denotes parity one women with zero completed years in parity one at beginning of year, "1" parity one women with one completed year in parity one at beginning of year, and so on.

Model Rates: Input to calculation. This schedule of rates plays a role analogous to that of the standard lx schedule in the Brass logit system.

ACUM1: Ascending cumulative. Gives cumulative progression to next birth implied by model rates. Calculated as 1--(1-rate) where the product is taken over the first i rates.

BID1: Birth interval distribution implied by model rates, calculated by differencing preceding column.

BID2: Defective birth interval distribution corresponding to given parity progression ratio, calculated by multiplying preceding column by constant chosen so that BID2 values sum to given parity progression ratio.

DCUM2: Descending cumulative. Gives proportions of women remaining in parity one at given completed years duration in parity one implied by birth interval distribution in preceding column. Calculated by cumulative substraction of values in preceding column from one.

Output Rates: Calculated by dividing values in BID2 column by values in DCUM2 column. This schedule of rates yields the parity progression ratio inputed to the calculation. As a check, one calculates 1 - (1-.0042)(1-.1431)...(1-.0115) to obtain 0.837.

Table 2. Calculating an Initial Open Birth Interval Distribution for Parity One women Given the Number of First Births in Year One and a Schedule of Parity Progression Rates

DIP	DCUM	Initial OBID
E	1.0000	123,469
0	.9958	122,950
1	.8533	105,356
2	.6203	76,588
3	.3890	48,029
4	.2733	33,744
5	.2154	26,595
6	.1865	23,027
7	.1721	21,249
8	.1649	20,360
9	.1630	20,125

Notes

Input to the calculation consists of the descending cumulative rates DCUM and the number of first births in the first year of the time series. Output of the calculation is the open birth interval distribution of parity one women at the beginning of the first year of the time series. In this case, the first year of the time series is 1975, and there were 123,469 first births registered in Taiwan during 1975. The DCUM values in this example are the DCUM2 values in Table 1. These values represent the proportions of women having first births in years before 1975 who are still in parity one at the beginning of 1975.

DIP: Same as Table 1.

DCUM: Descending cumulative distribution implied by given schedule of parity progression rates.

Table 3. Estimation of Period Parity Progression Ratios: Example of Progression from First to Second Birth for Taiwan in 1975 and 1976

		1975		1976	
DIP	Std Rates	[Births] OBID	Est Rates	[Births] OBID	Est Rates
E	.0042	[123,469]	.0040	[148,241]	.0047
0	.1431	122,950	.1352	122,965	.1604
1	.2731	105,356	.2581	106,327	.3062
2	.3729	76,588	.3524	78,164	.4180
3	.2974	48,029	.2810	49,598	.3334
4	.2119	33,744	.2002	34,533	.2375
5	.1342	26,595	.1268	26,988	.1504
6	.0772	23,027	.0730	23,223	.0865
7	.0418	21,249	.0395	21,346	.0469
8	.0115	20,360	.0109	20,410	.0129
9	0	20,125	0	20,138	0
		2nd Births			
	Proj	103,348	-	105,015	-
	Obs	97,661	-	117,725	-
	Ratio	0.9450	-	1.1210	-
Est PPPR		-	.817	-	.875

Notes

The standard rates and annual numhers of first and second births from 1975 are input to the calculation. The essential output are the estimated period parity progression ratios. Supplementary output includes the projected open birth interval distributions, the annual schedules of parity progression ratios, and the ratios of observed to projected births.

Std Rates: These are specific to the progression considered, first to second birth in this example. The values in this example are the output rates in Table 1.

[Births] OBID: The first entry in this column is the number of first births in 1975, for some of the women that have these births will contribute some (not many) second births in 1975. The remaining entries give numbers of parity one women at the beginning of 1975 at each completed years duration in parity one. These values come from Table 2.

Projected 2nd births are obtained by applying the rates in the 'Std Rates' column to corresponding values in the subsequent column and summing over all rows. This gives the number of 2nd births that would have occurred in 1975 if the observed rates were equal to the standard rates. The observed rates are estimated by multiplying the standard rates by the ratio of observed to projected 2nd births. These values are shown in the 'Est Rates' column. The estimated period parity progression ratio is

$$1 - (1-.0040)(1-.1352)(1-.0109).$$

The open birth interval distribution for the beginning of 1976 is obtained by projecting the distribution for the beginning of 1975 forward by one year, subtracting out the women who have a second birth during the year: e.g., 123,469x(1-.0040) = 122,965. This projected OBID is required for the next cycle of the estimation procedure, which i6 identical in form to the first cycle.