A New Interpretation of Brass' P/F Ratio Method Applicable When Fertility is Declining

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In the original P/F ratio method formulated by William Brass, children ever born data are used to adjust upward for under reporting numbers of births reported to have occurred during the year prior to a census or survey. The method assumes that (1) fertility has been constant, (2) the level of under reporting of births last year does not vary with age, and (3) children ever born for younger women are, if not completely reported, at least more completely reported than births during the prior year. On these assumptions, it is possible to compute an adjustment factor that inflates reported numbers of births last year to estimated true numbers.

For a presentation of the P/F ratio method see *Manual X: Indirect Techniques for Demographic Estimation*, Chapter II, Section B, pages 31-37, Population Studies No. 81, Department of International Economic and Social Affairs, United Nations, New York, 1983. This presentation explains in detail how estimates are computed and provides one application to well-behaved data, but it provides very little help with interpretation of problematic results. Nor does the rest of the demographic literature. The interpretation of problematic cases is the initial focus of these notes. The principal focus is a new interpretation of the P/F results that effectively eliminates the assumption of constant fertility.

The idea of the P/F ratio method is to compute what mean children ever born to women in each age group 15-19, 20-24, ... would be if fertility had been constant at the level indicated by the birth rates computed from the births last year data. These children ever born values are the "F" values to which the name of the method refers. The "P" values are the observed children ever born values. The calculation of the "F" values requires some interpolation and/or model calculations that may be done in various ways.

Because we expect births last year to be under reported, we expect the "F" values to be lower than the "P" values, so that the ratio P/F will be greater than one. The P/F ratio may be thought of as a correction factor that is applied to reported numbers of births last year, or to age-specific birth rates or total fertility rates calculated from these numbers of births, to estimate the corresponding true values.

P/F ratios are usually calculated for the 15-19, 20-24, ..., 45-49 age groups and the appropriate way of selecting an adjustment factor depends on the age pattern of these ratios. In considering the age pattern of the P/F ratios it is generally best to ignore the ratio for the 15-19 age group, for transformation that turns age-specific rates into mean children ever born may work rather poorly at the very beginning of the reproductive age span.

If age-specific birth rates have been exactly constant at some multiple (by a factor greater than one) of the rates computed from the births last year data, and if children ever born data is perfectly reported at all ages, the P/F ratios will be identical for all age groups.

If children ever born for women age 35 or older (say) are under reported, but the remaining assumptions of the preceding paragraph hold, the P/F ratios will still be identically equal for ages under 35 and the value for the younger age groups may be used to adjust the reported births last year upward.

If fertility has been declining but both the births last year and the children ever born data are completely reported, the "F" value for any age group will be lower than the "P" value because the "F" value represents current fertility, whereas the "P" value represents an amalgam of current and past fertility over the life of the cohort. If fertility decline has been occurring more or less continually over the life of the women represented, the divergence between the "F" and "P" values will increase with age and the P/F ratios will rise. In any case, the best P/F ratio to use for adjustment will be that for the youngest usable age group, 20-24.

If fertility has been declining and children ever born is completely reported for all age groups but births last year are under reported, the same rise in P/F ratios with age will be observed, but the overall level will be lower. In this case the P/F ratio for the 20-24 age group will be too low. Because the P/F ratios are increasing with age, it is likely that a ratio for one of the older age groups will be better. Unfortunately, however, with the traditional interpretation of the method, there is usually no way of deciding which P/F ratio to use. Under reporting of children ever born by older women will change the age pattern of the P/F ratios by pulling down the ratios for these older women, but this generally will not make it any easier to decide which P/F ratio to select.

The problems of interpreting P/F ratio results may be circumvented by adopting a new interpretation of the method. In the traditional interpretation, the problem is to find a way of adjusting births last year data for under reporting and the solution is to derive an adjustment factor by comparing these data with children ever born data. The constant fertility assumption comes in as a necessary condition for effecting the comparison. We need to assume constant fertility to infer children ever born (the "F") values from the births last year data.

In the new interpretation, nothing is assumed about fertility change. The problem is to estimate the completed fertility for each cohort represented, the cohorts aged 20-24, 25-29, ... at the time of the census or survey, completed fertility meaning the average number of children ever born women have had by the end of the reproductive age span.

If we know the age pattern of age-specific fertility rates for the cohort, we can compute the mean number of children ever born for any age group, and therefore the ratio of completed fertility to mean children ever born for any given age group. Thus if we know both (1) the mean number of children born to women aged 25-29 at a given point in time, and (2) the age-pattern of fertility for this cohort, we can compute it's completed fertility.

We estimate the age pattern of fertility for each cohort by the age-specific rates computed from the births last year data and compute an adjustment factor that translates mean children ever born for this age group into mean children born by the end of the reproductive age span. This adjustment factor may be denoted TFR/F(i), where TFR denotes the total fertility rate computed from the age-specific birth rates based on the (unadjusted) births last year data and F(i) denotes the mean children ever born for the i-th age group based on these same rates. For this ratio to be correct it is necessary only for the pattern, not the level, of the age-specific rates to be correct, i.e., as in the traditional P/F ratio method, we must assume that there is no differential under reporting of births last year, but not that reporting is complete.

To estimate completed fertility for the cohort we apply this adjustment factor to the children ever born value for the cohort, i.e., we calculate [TFR/F(i)]P(i). But this equals TFR[P(i)/F(i)], which is the P/F ratio for the age group times the total fertility rate implied by the unadjusted births last year data, i.e., is the estimate of total fertility that would result, in the traditional interpretation, if we selected this P/F ratio for the final estimate. In the new interpretation, however, these different TFR values are not conflicting estimates for the same value, but estimates of completed fertility for different cohorts.

The final element of the new interpretation is to translate these cohort fertility indicators into period indicators using the method of "time plotting" children ever born data. Time plotting exploits a correspondence between period and cohort measures developed long ago by Norman Ryder, who noted, in particular, that the mean number of children born to a cohort approximates the period total fertility rate at the time this cohort was at its mean age at childbearing. An early publication with a succinct statement of the problem is N. B. Ryder, "The Process of Demographic Translation," *Demography* **1** (1964):74-82; unfortunately, according to the author, this paper is full of misprints that makes the technical presentation nearly unreadable. A more recent exposition is given in Norman B. Ryder, "Cohort and Period Measures of Changing Fertility," in Rodolfo A. Buletao and Ronald D. Lee, eds., *Determinants of Fertility in Developing Countries*, Volume 2, Chapter 20, pages 737-756 (New York: Academic Press, 1983). Another useful source, whose appendices contain numerous derivations, is "Components of Temporal Variations in American Fertility," in R. W. Hiorns, ed., *Demographic Patterns in Developed Societies* (London: Taylor & Francis, Ltd., 1980).

"Time plotting" children ever born data for post-reproductive age women consists simply of figuring out when the cohort in question reached its mean age at childbearing and plotting mean children ever born for the cohort at this point in time. Concerns about completeness of reporting of children ever born by older women are best addressed by constructing two or more such time plots and examining them for evidence of inconsistency. Time plotting was introduced in Griffith Feeney, "The Demography of Aging in Japan: 1950-2025," NUPRI Research Paper Series No. 55, February 1990, Nihon University Population Research Institute, Tokyo, Japan, and is discussed as well in Griffith Feeney, "Child Survivorship Estimation: Methods and Data Analysis", *Asian and*

Pacific Population Forum **5**(Nos. 2-3, Summer-Fall 1991). A more complete, if relatively informal, discussion is provided in notes for a Notestein Seminar given at the Princeton University Office of Population Research in November 1995, "The Analysis of Children Ever Born Data for Post-Reproductive Age Women."

Mean age at childbearing does not usually vary a great deal between populations or over time within populations, and in most cases it will suffice to take the mean age at childbearing to be 30 years. Assuming this value, completed cohort fertility for cohorts aged 20-24, 25-29, 30-34, ... at time t estimates the period TFR at times t + 7.5 years, t + 2.5 years, t - 2.5 years, and so on., t denoting the time of the census or survey.

A convenient consequence of taking mean age at childbearing to be 30 years is thus that current fertility is estimated as the average of completed fertility for the 25-29 and 30-34 cohorts. More generally, the estimates for all age groups may be plotted against the corresponding time points to give an estimated trend of fertility.

Consequences of the reinterpretation of the P/F ratio method are that (1) we estimate a time trend of fertility rather than only the current level and (2) we obtain estimates of future as well as current and past fertility. Under reporting of children ever born data will give estimates for older age groups (earlier times) that are too low in precise correspondence with the degree of under reporting. The estimate of current fertility will be affected only by under reporting of children ever born by women under age 35.

Under reporting of children ever born data aside, there are two principal considerations that limit the accuracy of the results, the precision of the dating and the assumption that all cohort fertility patterns may be approximated by the births last year data. The precision of the dating is not likely to be an important factor unless fertility is changing rapidly and/or high precision is required. Mean age at childbearing does not vary greatly between populations, and even an error of two years in the dating will usually change levels by only a few tenths of a child per woman.

It should be noted in this connection, however, that there is an inherent limitation in using cohort data to estimate period data, in that cohort data are essentially 'smoothed' period data. Period estimates from cohort data are thus incapable of identifying sharp period fluctuations in level. If fertility levels are fluctuating sharply—not an expected condition in most situations where the P/F ratio method is applied—estimation from cohort data will give results similar to what would be obtained by smoothing the period figures.

Estimating cohort age patterns of fertility by the births last year data raises more complicated issues. Fertility decline means relatively fewer higher order births, and this implies a change in age pattern. Other things being equal, mean age at childbearing will decline as fertility declines, though this is often offset by rising age at marriage and age at first birth. Definitive conclusions on the robustness of estimates against departures from this assumption would require detailed simulation calculations.

In practice, a quick indication of how serious estimation errors may be will be obtainable by applying the procedure to two or more successive data sources and seeing how well the time series from the two sources match up.