

# Lexis Diagrams

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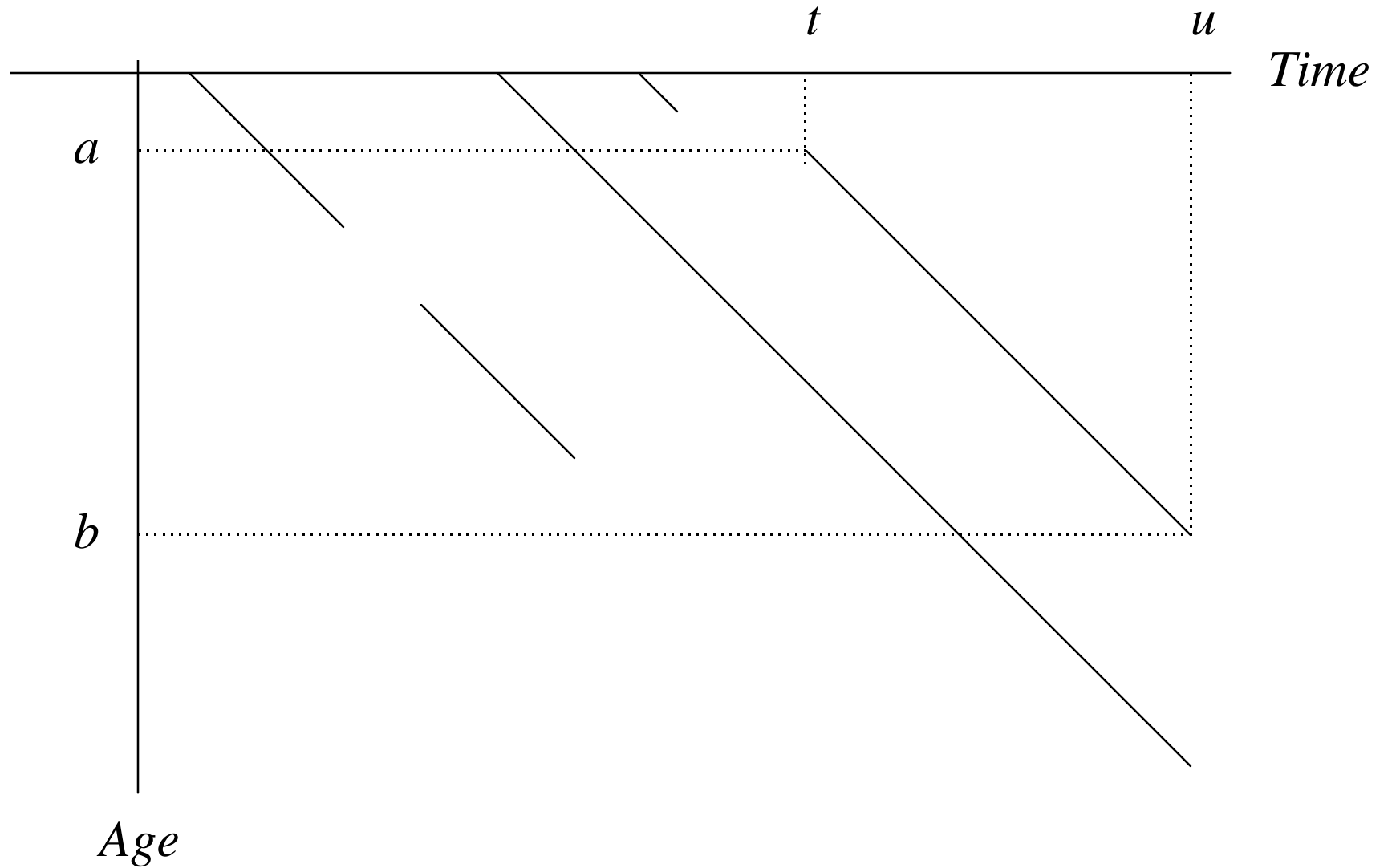
# Lexis Diagrams

- represent relationships between sets of persons and events
- are a specialized, highly effective visual language
- like any other language, require study *and practice* for effective use

# Life Lines

- The history of any individual's membership in a population is represented graphically on coordinate axes by a straight line or broken straight line segment which begins at the age and time of this individual's first entry to the population and ends at the time and age of last exit from the population. This line is called a **life line**.

# Life Lines



# Repeated Entry and Exit

- The life line of an individual with a single spell of membership in a population is a single straight line segment.
- The life line of an individual who enters and exits the population more than once is a broken straight line segment, each segment representing a spell of membership.

# Life Lines Not (Usually) Drawn

- Except occasionally for illustrative purposes, life lines for actual persons are not drawn, only imagined.
- Imagined life lines form the basis for visualizing sets of persons as described below.

# The Age-Time Plane

- This half plane is called the **age-time plane**.
- The age axis is drawn down because tables showing statistics by age show the youngest age in the first row with older ages in subsequent rows.
- Other orientations (age axis going up, age and time axes reversed) are sometimes found.

# Referencing Points and Lines on the Age-Time Plane

- Points are referenced by their time and age coordinates, e.g.,  $(t, a)$  for the point at time  $t$  and age  $a$
- Lines are referenced by their endpoints, e.g., the line connecting the points  $(t, a)$  and  $(u, b)$
- Broken life lines are referenced by their constituting segments



# Exact Age and Age in Completed Years

- A person's **exact age** at any given time is the time elapsed since this person's birth.
- A person's **age in completed years** at any given time is the greatest integer less than this person's exact age.
- Age in completed years is also referred to as **age at last birthday**.

# Time and Time Periods

- **Time** refers to a point in time and is specified by writing, e.g., 1984.754, with the appropriate number of places after the decimal point being determined by context.
- **Time period** refers to an interval beginning and ending at specified times.
- Time and time period are analogous to exact age and age group.

# Dates and Times

- **Dates**, expressed as day, month, and year, represent time *periods*, not times.
- For many purposes, time specified within a 24 hour period is sufficient precision.
- When translating dates to times, some convention, e.g., mid-day, is needed.

# Decimal Translation of Dates

- To calculate the fraction of a year (365 days) represented by a given date, add the days in months preceding the given month to the given day minus one half and divide by the number of days in the year.
- Three decimal places (e.g., 1980.123) gives precision to the day and allows inversion.

# Calendar Years and Census Years

- **Calendar years** begin at 0000 hours on January 1 and end at 2400 hours on December 31.
- **Census years** begin and end at the reference time of a specified census.

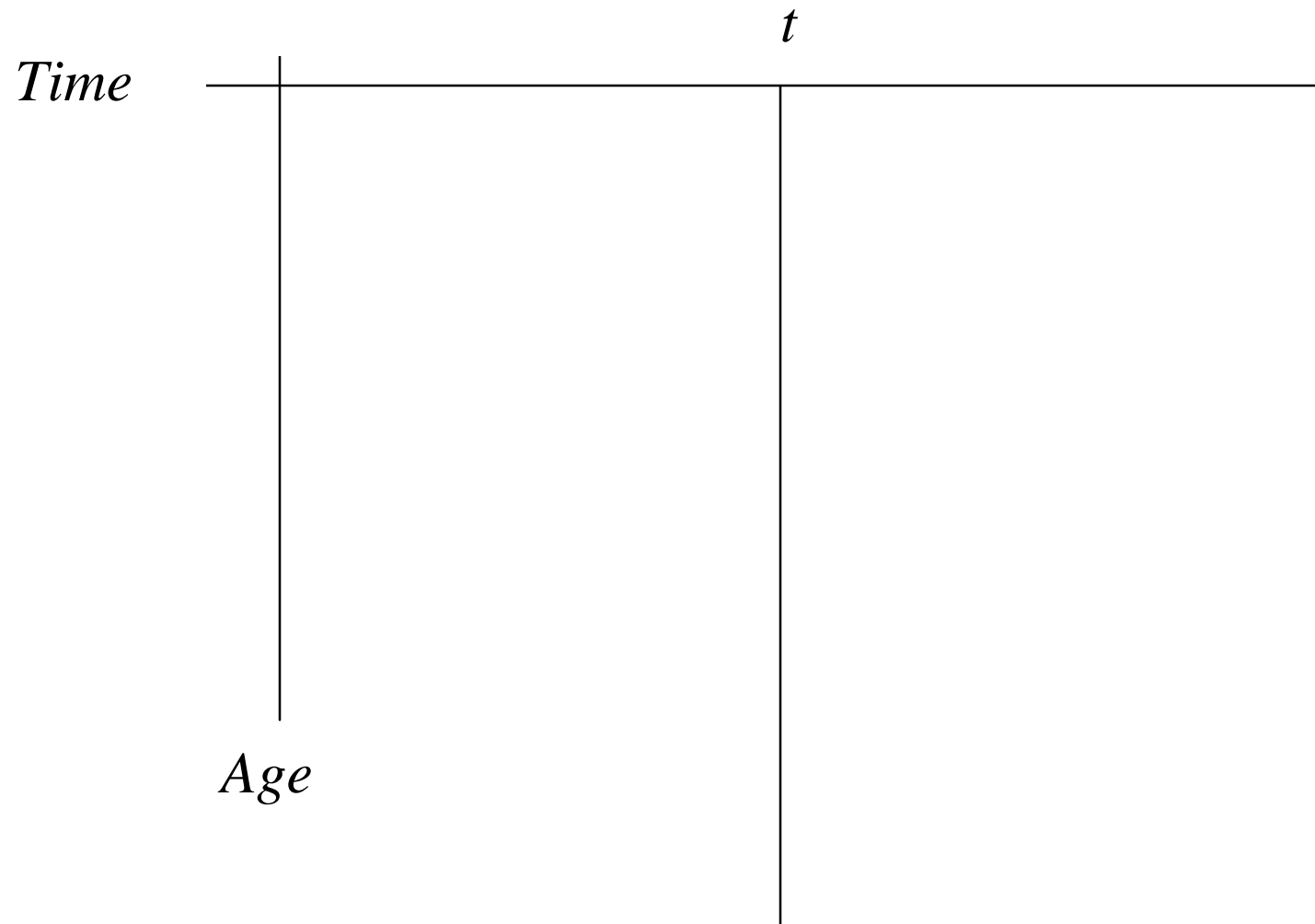
# Visualizing Sets of Individuals

- Any straight line segment in the age-time plane represents the set of individuals whose life lines intersect this line.
- This is the Lexis diagram **representation principle for sets of individuals**.
- The principle is general, but practical uses involve only a few special cases.

# Individuals in population at time $t$

- The half line perpendicular to the time axis and intersecting it at time  $t$  represents the set of all individuals in the population at time  $t$ .
- Because this is the set of individuals who should be enumerated by a census at time  $t$ , it is sometimes called the **census set** of the population at time  $t$

# Individuals in population at time $t$

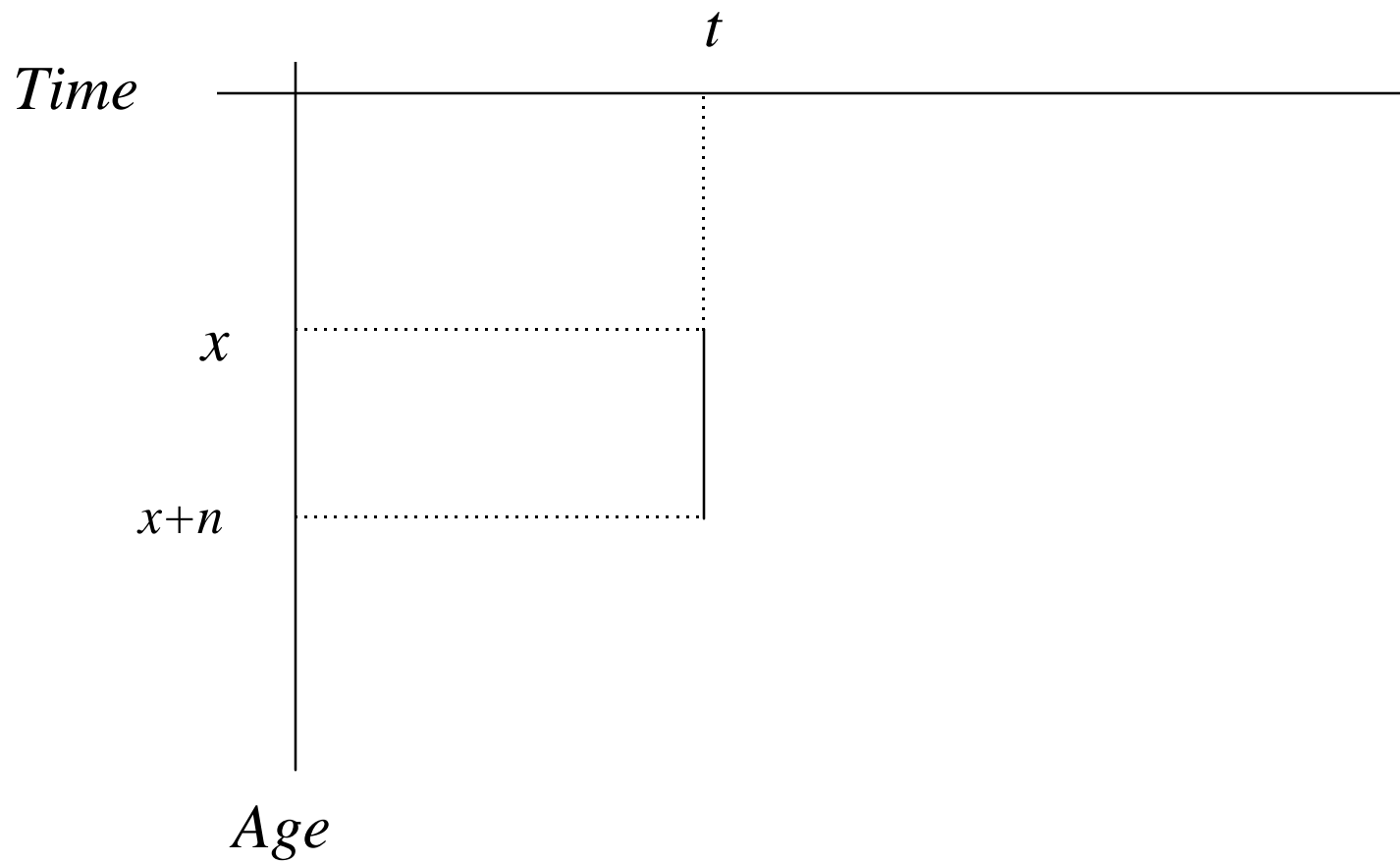




## Individuals aged $x$ to $x+n$ at time $t$

- The straight line connecting the points  $(t, a)$  and  $(t, a+n)$  represents the set of all individuals in the population at time  $t$  who are aged  $a$  to  $a+n$  at time  $t$ .
- Partitioning the half line representing total population at time  $t$  into segments corresponds to distributing the population at time  $t$  by age group.

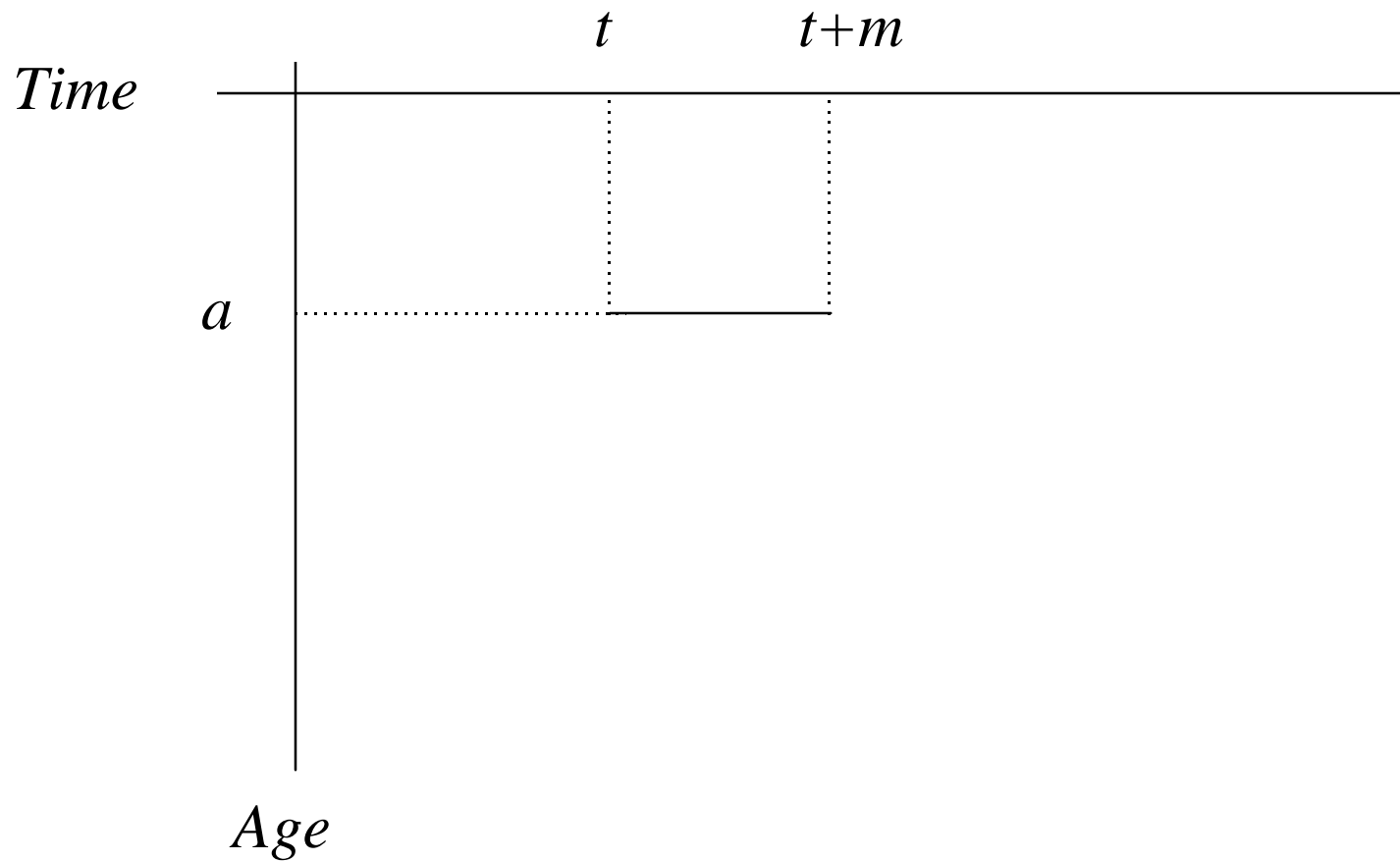
# Individuals aged $x$ to $x+n$ at time $t$



## Individuals attaining age $x$ between times $t$ and $t+n$

- The line connecting the points  $(t, a)$  and  $(t+n, a)$  represents the set of all individuals in a population who reach exact age  $a$  between time  $t$  and time  $t+n$ .
- In the special case  $a = 0$ , this line represents births to the population during time  $t$  and time  $t+n$ .

# Individuals attaining age $x$ between times $t$ and $t+n$



# EXERCISE 1

Draw a Lexis diagram indicating:

- all persons in a population at (time) 1980;
- persons aged 0-4 at time  $t$ ;
- persons aged 5-9 at time  $t+5$ ;
- persons reaching exact age 5 between time  $t$  and time  $t+5$ ;
- how are the last three sets related? (Hint: draw some illustrative life lines.)

# Representing Deaths

- A death is represented on the age-time plane by a point whose coordinates  $(t, a)$  are the time  $t$  at which the event occurred and the age  $a$  of the individual to whom it occurred.
- The point representing a death lies at the end of an individual's life line.
- Points representing actual deaths, like life lines, are (usually) imagined, not drawn.

# Visualizing Sets of Deaths

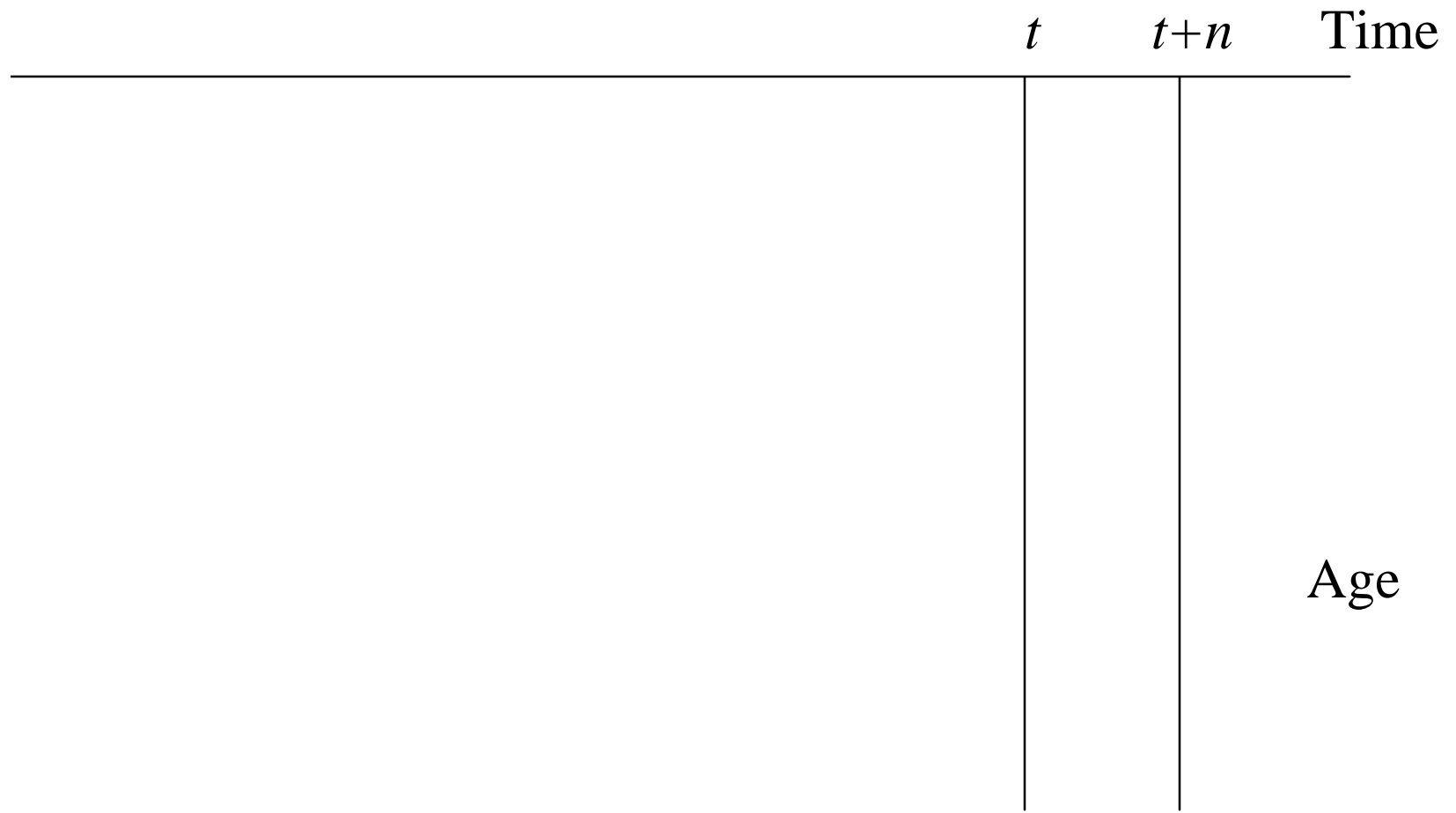
- Any two dimensional set in the age-time plane represents the set of deaths represented by points within it.
- This is the Lexis diagram **representation principle for deaths**.
- The principle applies to *any* two dimensional set, but most applications involve a few special cases.

## Deaths between times $t$ and $t+n$

- Deaths to a population between time  $t$  and time  $t+n$  are represented by the open-ended rectangle bounded by the time axis and the perpendiculars to the time axis at times  $t$  and  $t+n$ .
- The same set in the age-time plane may represent events of any other type, e.g., births, migrations, emigrations.



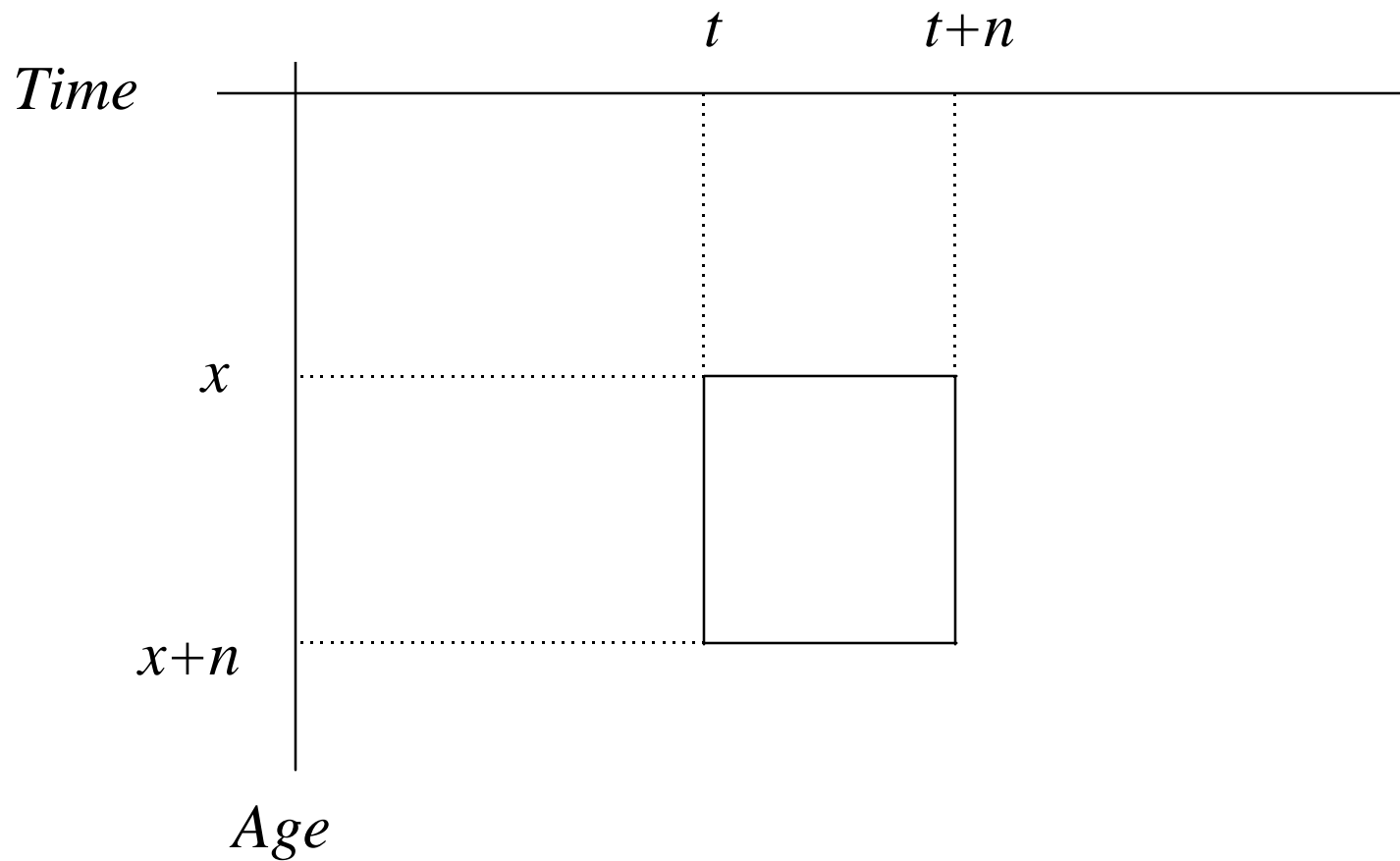
# Deaths between times $t$ and $t+n$



## Deaths to individuals aged $x$ to $x+n$ between times $t$ and $t+m$

- The set of deaths occurring in a population between times  $t$  and  $t+m$  to persons aged  $x$  to  $x+n$  at the time of death is represented by the rectangle formed by the points  $(t,x)$ ,  $(t+m,x)$ ,  $(t,x+n)$ , and  $(t+m,x+n)$ .
- The disaggregation of deaths by age is represented by partitioning the vertical strip at  $(t,t+m)$  with lines at age group breaks.

# Deaths to individuals aged $x$ to $x+n$ between times $t$ and $t+m$



## EXERCISE 2

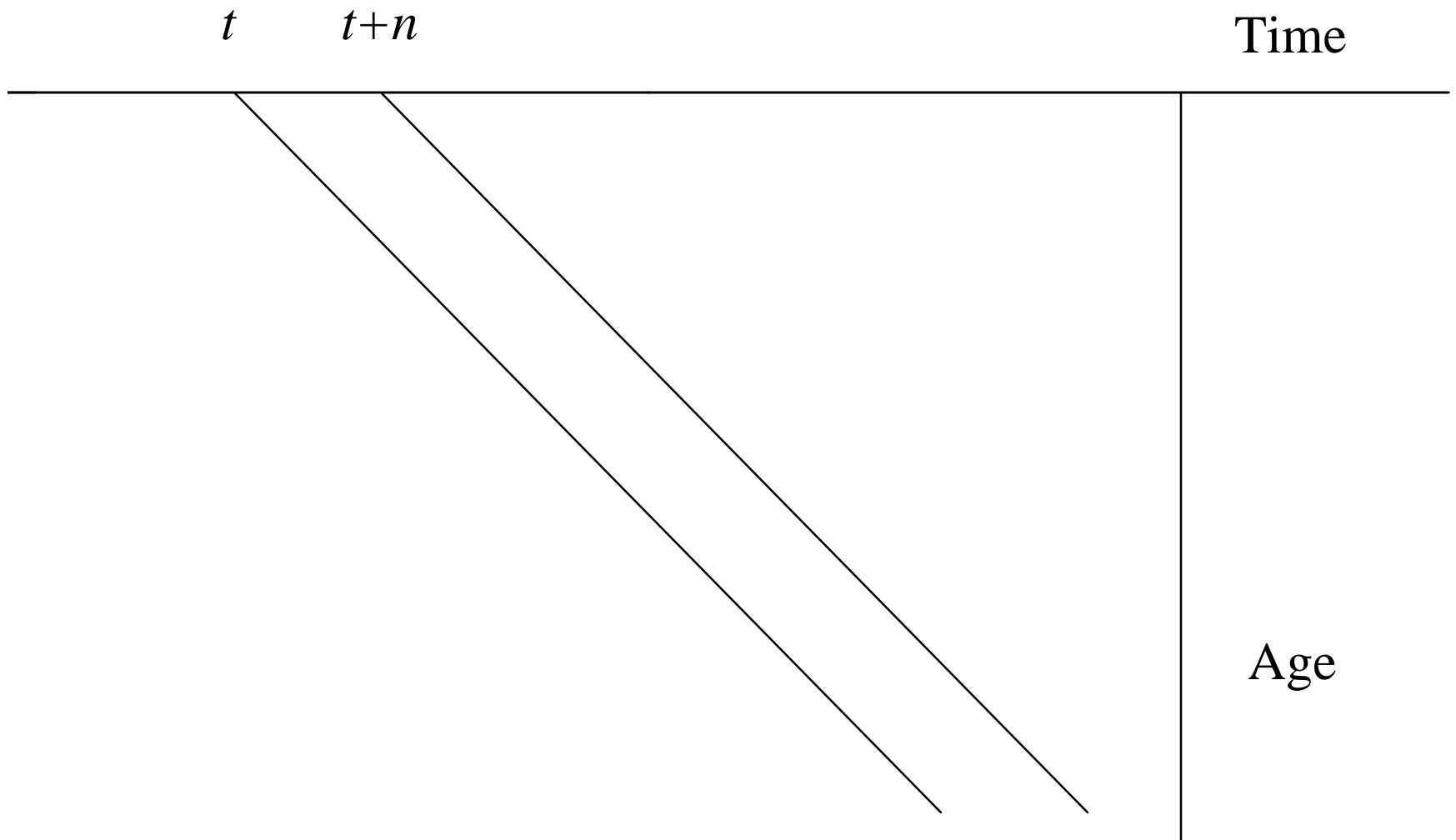
Draw a Lexis digram indicating:

- deaths to a population during 1980-1984;
- deaths by 5 year age group through 20-24;
- deaths in the open-ended age group 25+;
- deaths during 1980-1984 to individuals born during 1980-1984;
- deaths during 1980-84 to persons 25 and over in 1980.

## Deaths of individuals born between times $t$ and $t+m$

- The set of all deaths to members of the birth cohort born between times  $t$  and  $t+m$  is represented by the open-ended diagonal strip bounded by the line connecting the points  $(t, 0)$  and  $(t+m, 0)$  and the diagonal lines extending down and to the right from the endpoints of this line.

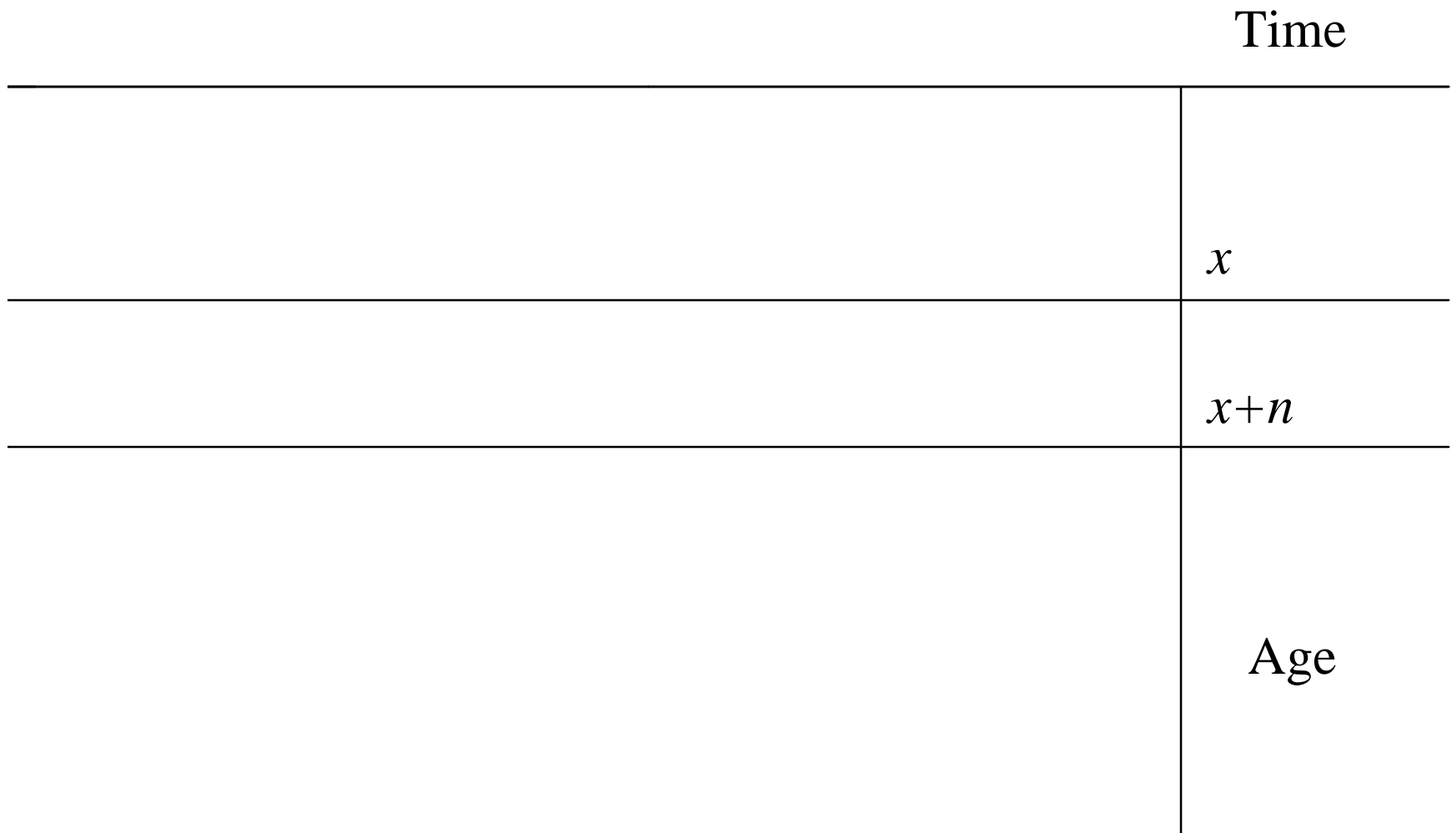
# Deaths of individuals born between times $t$ and $t+m$



# Deaths of individuals aged $x$ to $x+n$ at the time of death

- The set of all deaths of individuals aged  $x$  to  $x+n$  at the time of death is represented by the doubly open-ended horizontal strip bounded by the lines that are perpendicular to the age axis and pass through the points  $x$  and  $x+n$

# Deaths of individuals aged $x$ to $x+n$ at the time of death





# Lexis Diagram Translation

- Working with Lexis diagrams requires two way translation.
- We need to translate verbal descriptions into Lexis diagram representations.
- We need to translate Lexis diagram representations into verbal descriptions.
- There are two general translation methods.

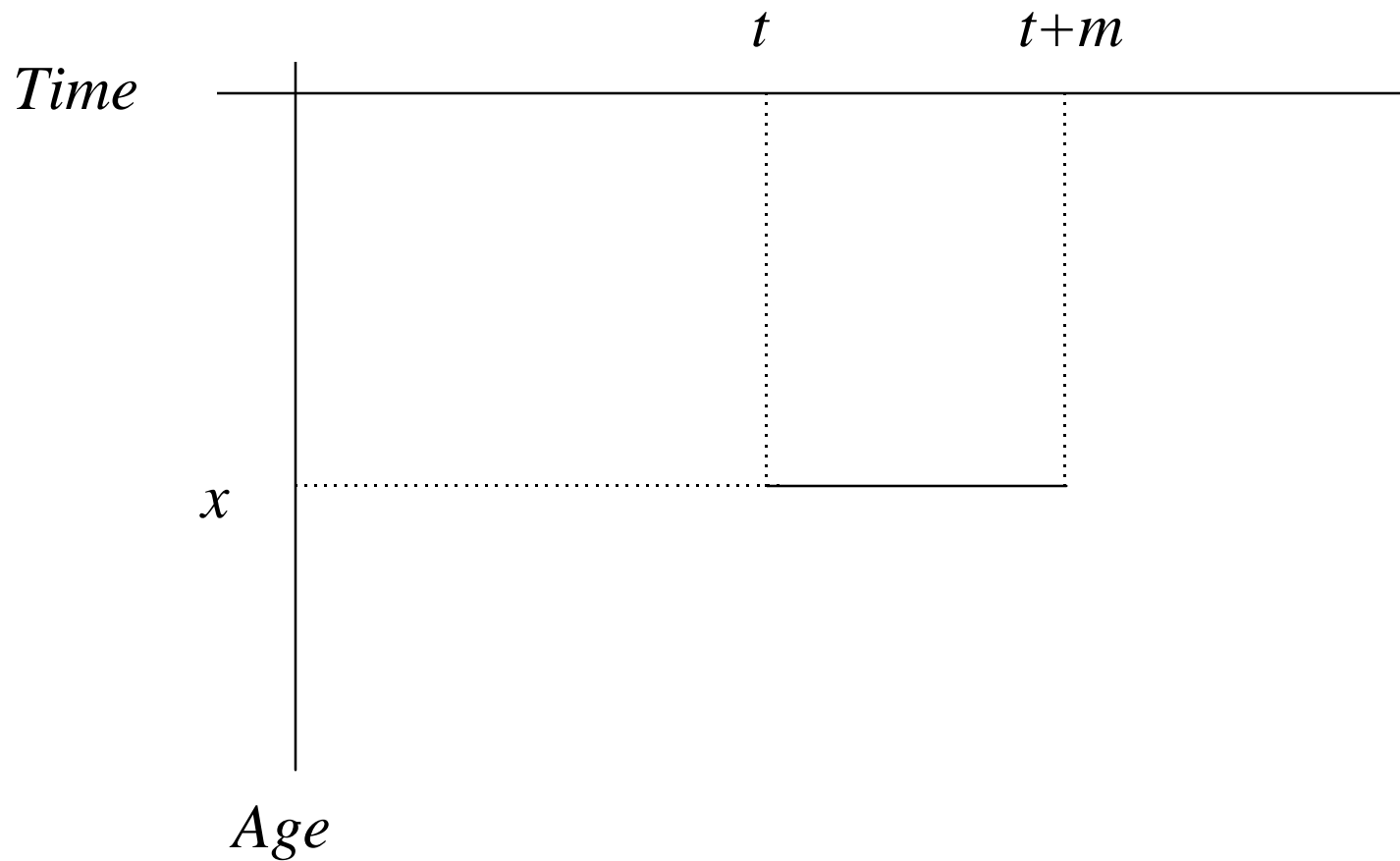
# The Method of Extremes

- Applies to sets of persons and deaths.
- Identify the extreme possibilities consistent with the description; each extreme will be a point in the age-time plane.
- Draw lines connecting these points to form a line segment (set of individuals) or plane figure such as a rectangle, parallelogram, or triangle (set of events).

# Individuals reaching exact age $x$ between times $t$ and $t+m$

- There are two extremes, reaching age  $x$  at time  $t$  exactly and reaching age  $x$  at time  $t+m$  exactly.
- The corresponding points are  $(t,x)$  and  $(t+m,x)$ .
- The line connecting these points is the Lexis diagram representation.

# Individuals reaching exact age $x$ between times $t$ and $t+m$



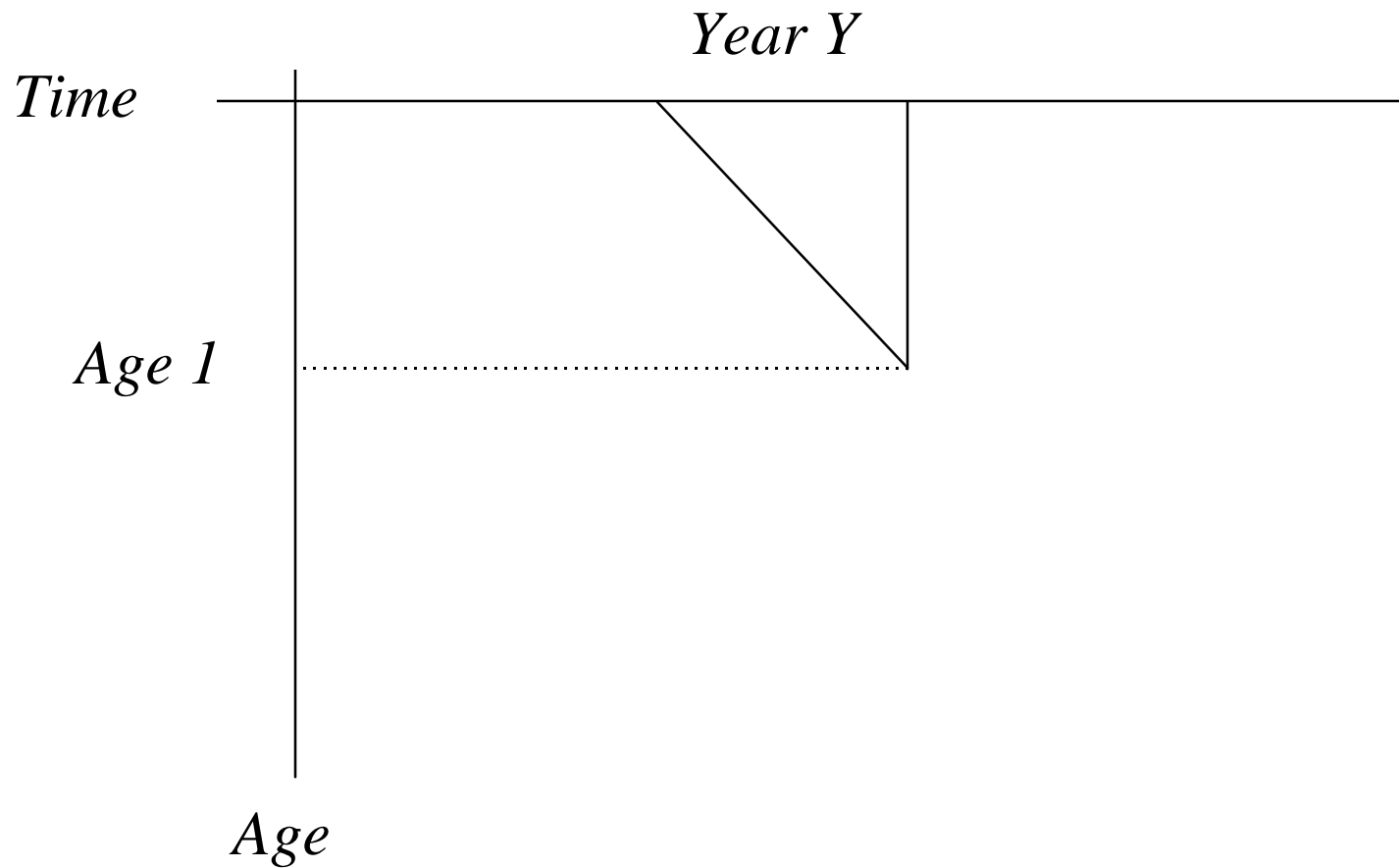
# Infant deaths during year $Y$ to children born during year $Y$

- There are three extreme possibilities corresponding to birth and death at the beginning and end of the year.
- Born at beginning of year, die immediately
- Born at beginning of year, die at end of year
- Born at end of year, die immediately
- Question: Why not four possibilities?

## *(Continued)*

- The points corresponding to these three extreme cases are ( $y$  denotes beginning of year  $Y$ ):
- point  $(y, 0)$ , point  $(y, 1)$ , and point  $(y+1, 0)$
- The desired representation is the triangle formed by the lines connecting these three points (draw it!)

# Infant deaths during year $Y$ to children born during year $Y$



## EXERCISE 3: Draw a Lexis diagram representing:

- Deaths during 1980-1984 to persons aged 0-4 at the beginning of 1980.
- Deaths during 1980-1984 to persons aged 0-4 at the beginning of 1980 who were aged 0-4 at the time of death.
- Deaths during 1980-1984 to persons aged 0-4 at the beginning of 1980 who were aged 5-9 at the time of death.



*(continued)*

- What is the relationship between these three sets of deaths?
- What is the relationship between these three sets of deaths and the sets of persons represented in Exercise 1?

# The Method of Intersections

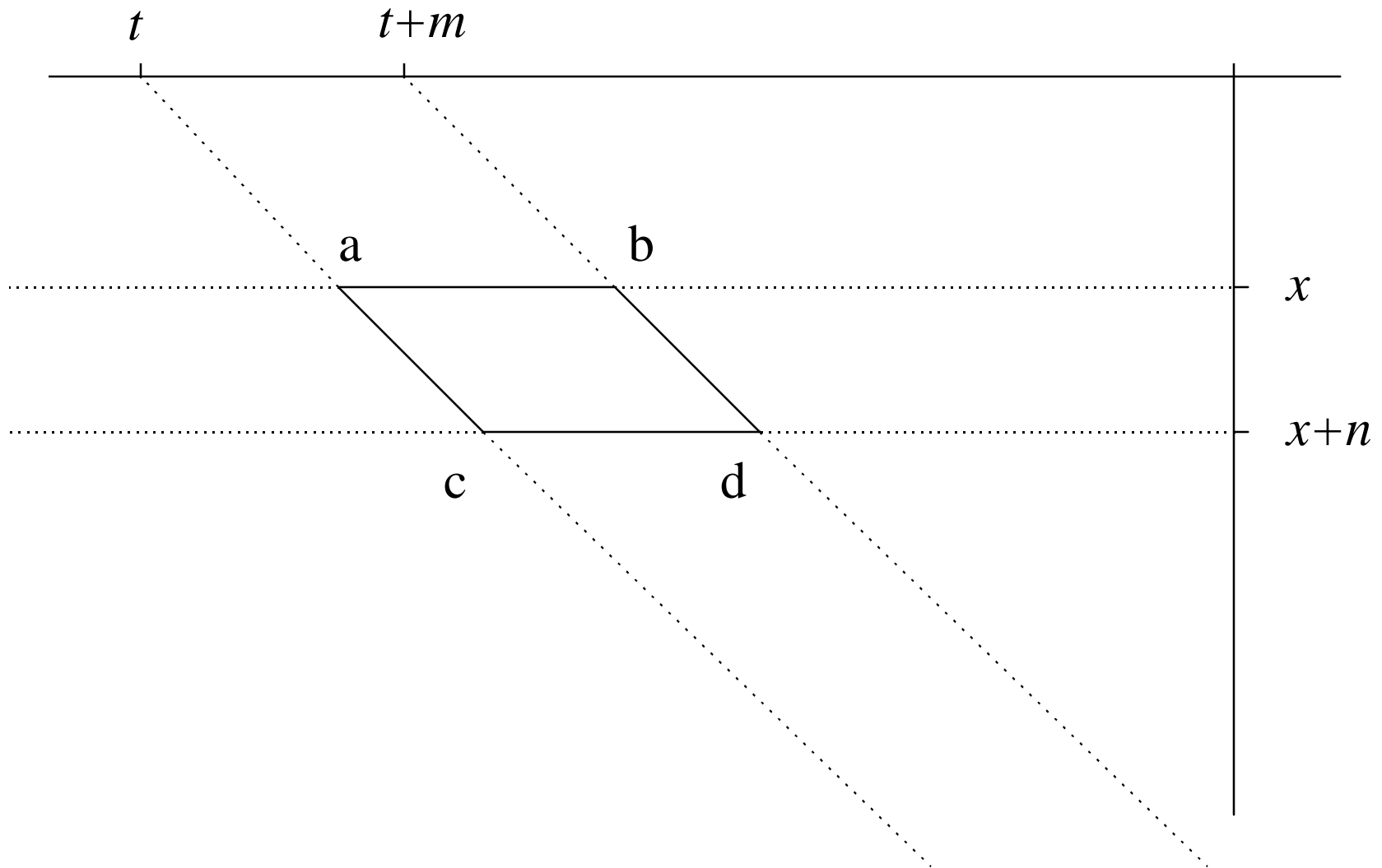
- Does not apply to sets of individuals.
- Identify references to time period, age group, and/or birth cohort.
- Draw the corresponding vertical, horizontal, or diagonal strips.
- The Lexis diagram representation is the intersection of these strips.

# Method of Intersections

## Example 1

- Draw the representation of the set of deaths occurring at ages  $x$  to  $x+n$  to individuals born between times  $t$  and  $t+m$ .
- For ease of reference, label points on diagram and refer to figures formed by given points, e.g., parallelogram  $abcd$ .

*(continued)*



# EXERCISE 4

Draw Lexis diagram representations of

- Deaths between times  $t$  and  $t+m$  to persons aged  $x$  to  $x+n$  at time  $t$ .
- Deaths during year  $Y$  to individuals aged  $x$  in completed years at the beginning of year  $Y$  that occur to individuals aged  $x+1$  in completed years at the time of death.

# Generalization 1:

## Other Classes of Events

- Everything said thus far about representing sets of deaths applies to any other class of events, births, migrations, marriages, divorces, IUD insertions, and so on.
- The point representing any event must lie on the life line of the individual who experiences the event.
- The class of events must be identified.

## Generalization 2: Other Duration Variables

- Age is defined as time elapsed since birth.
- **Duration variables** measure time elapsed since occurrence of a specified event, such as first marriage, birth of a child, or insertion of an IUD.
- The age axis in the Lexis diagram may be replaced by an axis representing any duration variable.

# Example 1

## Ever Married Women

- Consider the subpopulation of a given population that consists of ever married women.
- Women enter this population by marrying for the first time and leave it by dying.
- Lexis diagrams for this population may show either age or duration of marriage as the non-time axis.



# Example 2

## Parity One Women

- Consider the subpopulation of a given population that consists of women with exactly one child.
- Women enter this population by having a first birth and leave it either by having a second birth or by dying.
- Lexis diagrams may show either age or **open birth interval** as the non-time axis.

The End